



Adapting Non-Standard Weighted Average Approach for Solving the Lotka-Volterra Model

Mohammad Khatim Hasan* and Shahrizan Mazlan

Center for Artificial Intelligence Technology, Faculty of Information Science and Technology, Universiti Kebangsaan Malaysia, Malaysia

ABSTRACT

Simulating Lotka-Volterra model using a numerical method requires the researcher to apply tiny mesh sizes to come up with an accurate solution. This approach will increase the complexity and burden of computer memory and consume long computational time. To overcome these issues, a new solver is used that could simulate Lotka-Volterra model using bigger mesh size. In this paper, prey and predator behaviour is simulated via Lotka-Volterra model. We approximate the nonlinear terms in the model via weighted average approach and differential equation via nonstandard denominators. We provide three new schemes for one step method and simulate four sets of parameters to examine the performance of these new schemes. Results show that these new schemes simulate better for large mesh sizes.

Keywords: Lotka-Volterra, Nonlinear, Nonstandard method, Weighted average approach.

INTRODUCTION

Lotka-Volterra model represents a classical mathematical interaction of prey and predator. The model was proposed by Alfred Lotka and Vito Volterra in 1926 in two separate researches (Anisiu, 2014). In the model, at

least two variables represent the prey and the predators presented in at least two differential equations. Prey is assumed to have unlimited source of food, while the only source of food for the predators is the prey. This model is also used in business transactions (Ye et al., 2013), transportation (Yuting & Meng, 2011), security (Yang & Chen, 2015) and many other applications. Ye, Qiang, and Song (2013) for example applied Lotka-Volterra model in e-business to predict online transaction behaviour. At the same time, Obaid, Ouifki, and Patidar (2013) used Lotka-Volterra to model HIV infection mathematically. Yuting and Meng (2011) used the Lotka-Volterra model to analyse highway user and train user

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E-mail addresses:

mkh@ukm.edu.my (Mohammad Khatim Hasan),

shahrizan.mazlan@yes.my (Shahrizan Mazlan)

*Corresponding Author

while Yang and Chen (2015) developed a fire security system to educate awareness of fire security level in university using Lotka-Volterra model.

This study was aimed at improving the accuracy of existing numerical approach for big mesh size since numerical method requires researchers to apply tiny mesh sizes (Zibaei & Namjoo, 2016) to generate accurate solutions but sometimes the latter do not converge (Obaid, Ouifki, & Patidar, 2013). In this paper, the authors applied nonstandard approximation and weighted average approach to solve the problem. Specifically, the study used Mickens (2003), and Yaacob and Hasan (2015) nonstandard approach. As a contribution, we adopted the weighted average (Sejong, Jimmie, & Yongdo, 2011) concept to represent nonlinear terms in Lotka-Volterra model.

MATERIALS AND METHODS

A Lotka-Volterra model with two equations is given below:

$$\frac{dx}{dt} = Ax - Bxy, \quad \frac{dy}{dt} = -Cy + Dxy, \tag{1}$$

where x and y denote prey and predator population respectively; $A > 0$ represents prey birth rate, $B > 0$ represents prey captured by predator rate, $C > 0$ represents predator death rate and $D > 0$ is predator's birth rate.

In this section, three new one-step methods were proposed and derivation of all three schemes are discussed. We approximate $\frac{dx}{dt}$ and $\frac{dy}{dt}$ following Mickens (2003), Yaacob and Hasan (2015), and Bhowmik (2009). The approximation is given by

$$\frac{dx}{dt} \approx \frac{x_{i+1} - x_i}{\emptyset},$$

$$\frac{dy}{dt} \approx \frac{y_{i+1} - y_i}{\emptyset}.$$

Where $\emptyset = \tan(h)$. Scheme 1 is developed by replacing x in eq. (1) with a weighted average (Sejong, Jimmie, & Yongdo, 2011), which was different from other studies on nonstandard schemes (Mickens, 2003; Obaid, Ouifki, & Patidar, 2013; Yaacob & Hasan, 2015; Zibaei & Namjoo, 2016)

$$x \rightarrow wx_i + (1 - w)x_{i+1}, \rightarrow x_{i+1}y_i, \text{ thus}$$

$$\frac{dx}{dt} = Ax - Bxy \rightarrow \frac{x_{i+1} - x_i}{\emptyset} = A(wx_i + (1 - w)x_{i+1}) - Bx_{i+1}y_i$$

$$x_{i+1} - x_i = A\emptyset wx_i + A\emptyset x_{i+1} - A\emptyset wx_{i+1} - B\emptyset x_{i+1}y_i, \tag{2}$$

$$x_{i+1} - A\emptyset x_{i+1} + A\emptyset wx_{i+1} + B\emptyset x_{i+1}y_i = x_i + A\emptyset wx_i,$$

$$x_{i+1}(1 - A\emptyset + A\emptyset w + B\emptyset y_i) = x_i(1 + A\emptyset w),$$

$$x_{i+1} = \frac{1 + A\emptyset w}{1 - A\emptyset + A\emptyset w + B\emptyset y_i} x_i,$$

and $y \rightarrow wy_i + (1 - w)y_{i+1}$, $xy \rightarrow x_{i+1}y_i$, thus

$$\frac{dy}{dt} = -Cy + Dxy \rightarrow \frac{y_{i+1} - y_i}{\emptyset} = -C(wy_i + (1 - w)y_{i+1}) + D(x_{i+1}y_i)$$

$$y_{i+1} - y_i = -C\emptyset wy_i - C\emptyset y_{i+1} + C\emptyset wy_{i+1} + D\emptyset x_{i+1}y_i \tag{3}$$

$$y_{i+1} + C\emptyset y_{i+1} - C\emptyset wy_{i+1} = y_i - C\emptyset wy_i + D\emptyset x_{i+1}y_i$$

$$y_{i+1}(1 + C\emptyset - C\emptyset w) = y_i(1 - C\emptyset w + D\emptyset x_{i+1}),$$

$$y_{i+1} = \frac{1 - C\emptyset w + D\emptyset x_{i+1}}{1 + C\emptyset - C\emptyset w} y_i.$$

Eq. (2) and (3) represent scheme 1.

Scheme 2 is developed by replacing $xy \rightarrow 2x_{i+1}y_i - x_iy_i$ while other terms remain the same as in scheme 1, thus

$$\frac{y_{i+1} - y_i}{\emptyset} = -C(wy_i + (1 - w)y_{i+1}) + D(2x_{i+1}y_i - x_iy_i),$$

$$y_{i+1} - y_i = -C\emptyset wy_i - C\emptyset y_{i+1} + C\emptyset wy_{i+1} + 2D\emptyset x_{i+1}y_i - D\emptyset x_iy_i,$$

$$y_{i+1} + C\emptyset y_{i+1} - C\emptyset wy_{i+1} = y_i - C\emptyset wy_i + 2D\emptyset x_{i+1}y_i - D\emptyset x_iy_i \tag{4}$$

$$y_{i+1}(1 + C\emptyset - C\emptyset w) = y_i(1 - C\emptyset w + 2D\emptyset x_{i+1} - D\emptyset x_i),$$

$$y_{i+1} = \frac{1 - C\emptyset w + 2D\emptyset x_{i+1} - D\emptyset x_i}{1 + C\emptyset - C\emptyset w} y_i.$$

Eq. (2) and (4) represent scheme 2.

Scheme 3 is developed by replacing $xy \rightarrow 2x_{i+1}y_i - x_iy_{i+1}$, while others remain the same as in scheme 1, thus

$$\frac{y_{i+1} - y_i}{\emptyset} = -C(wy_i + (1 - w)y_{i+1}) + D(2x_{i+1}y_i - x_iy_{i+1}),$$

$$y_{i+1} - y_i = -C\emptyset wy_i - C\emptyset y_{i+1} + C\emptyset wy_{i+1} + 2\emptyset D x_{i+1}y_i - D\emptyset x_iy_{i+1},$$

$$y_{i+1} + C\emptyset y_{i+1} - C\emptyset wy_{i+1} + D\emptyset x_iy_{i+1} = y_i - C\emptyset wy_i + 2\emptyset D x_{i+1}y_i \tag{5}$$

$$y_{i+1}(1 + C\emptyset - C\emptyset w + D\emptyset x_i) = y_i(1 - C\emptyset w + 2D\emptyset x_{i+1}),$$

$$y_{i+1} = \frac{1 - C\emptyset w + 2D\emptyset x_{i+1}}{1 + C\emptyset - C\emptyset w + D\emptyset x_i} y_i.$$

Eq. (2) and (5) represent scheme 3.

The algorithm for scheme 1 is constructed by using approximate equation (2) and (3) and shown in Algorithm 3.1, while algorithm for scheme 2 is constructed using approximate equation (2) and (4) and shown in Algorithm 3.2, and for scheme 3 using (2) and (5) as shown in Algorithm 3.3.

In order to examine the performance of these new schemes, a numerical experiment with four sets of parameters (Prian, 2013) was conducted. The parameters are

- (1) $A = 0.4; B = 0.06; C = 0.12; D = 0.0006; h = 0.001, x_0 = 140, y_0 = 6, w = 0.37,$
- (2) $A = 0.4; B = 0.06; C = 0.4; D = 0.0006; h = 0.001, x_0 = 140, y_0 = 6, w = 0.37,$
- (3) $A = 0.4; B = 0.06; C = 0.74; D = 0.0006; h = 0.001, x_0 = 140, y_0 = 6, w = 0.37,$
- (4) $A = 0.4; B = 0.06; C = 0.74; D = 0.0006; h = 1.0, x_0 = 140, y_0 = 6, w = 0.37.$

Scilab code was developed to implement all algorithms and simulated results were compared with those t generated using Adam-Bashforth-Moulton's method (Hasan, Karim, S. A., & Sulaiman, 2015), which is order of accuracy.

<p>Algorithm 3.1: Algorithm for scheme 1</p> <p>Set</p> <p>$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$</p> <p>Calculate x and y ($i = 1, 2, \dots, n - 1$) using</p> $x_{i+1} = \frac{1 + A\phi w}{1 - A\phi + A\phi w + B\phi} x_i$ $y_{i+1} = \frac{1 - C\phi w + D\phi x_{i+1}}{1 + C\phi - C\phi w} y_i$ <p>Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. predator</p>	<p>Algorithm 3.2: Algorithm for scheme 2</p> <p>Set</p> <p>$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$</p> <p>Calculate x and y ($i = 1, 2, \dots, n - 1$) using</p> $x_{i+1} = \frac{1 + A\phi w}{1 - A\phi + A\phi w + B\phi} x_i$ $y_{i+1} = \frac{1 - C\phi w + 2D\phi x_{i+1} - D\phi x_i}{1 + C\phi - C\phi w} y_i$ <p>Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. Predator</p>
<p>Algorithm 3.3: Algorithm for scheme 3</p> <p>Set</p> <p>$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$</p> <p>Calculate x and y ($i = 1, 2, \dots, n - 1$) using</p> $x_{i+1} = \frac{1 + A\phi w}{1 - A\phi + A\phi w + B\phi} x_i$ $y_{i+1} = \frac{1 - C\phi w + 2D\phi x_{i+1}}{1 + C\phi - C\phi w + D\phi x_i} y_i$ <p>Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. predator</p>	

RESULTS AND DISCUSSION

Graphs plotted using Scilab code are shown in Figures 1-6. Figure 1 shows that all method produces exactly the same results. These show the new one-step schemes simulate comparable results with Adam-Bashforth-Moulton's method.

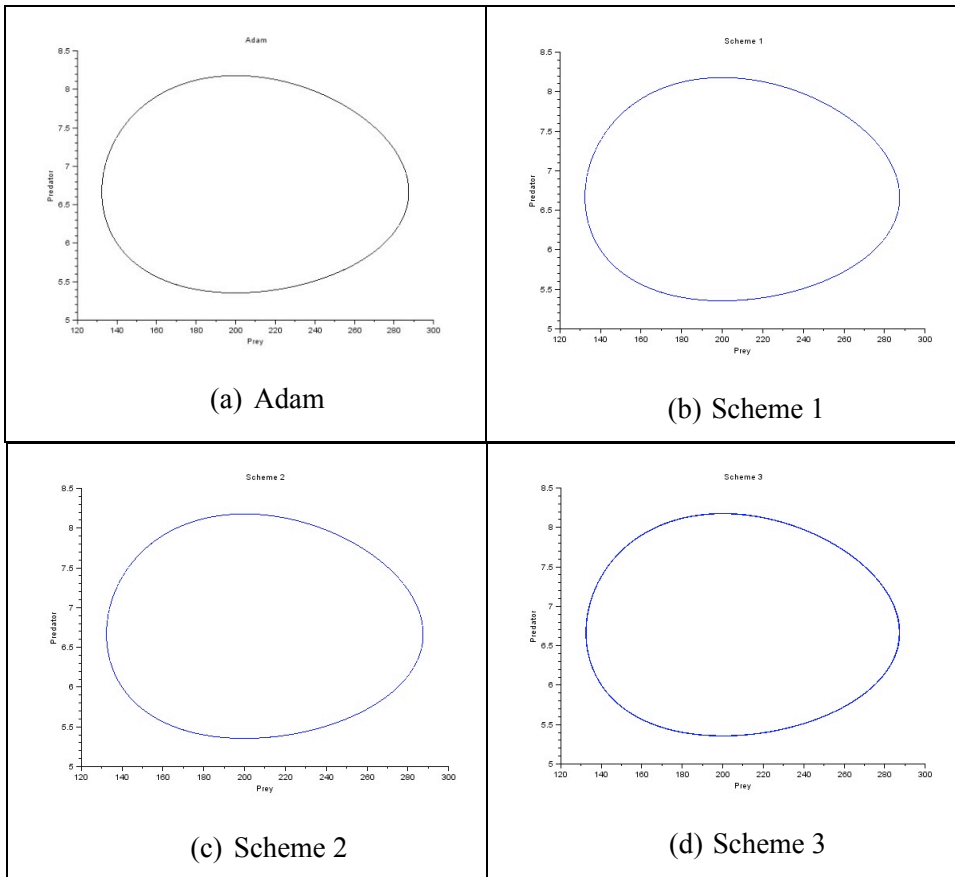


Figure 1. Prey vs predator for parameter set 1

Figure 2 shows scheme 1 and scheme 2 produced exactly the same result if one was using Adam's method. However, scheme 3 produced thicker result. The interaction value however, is almost similar.

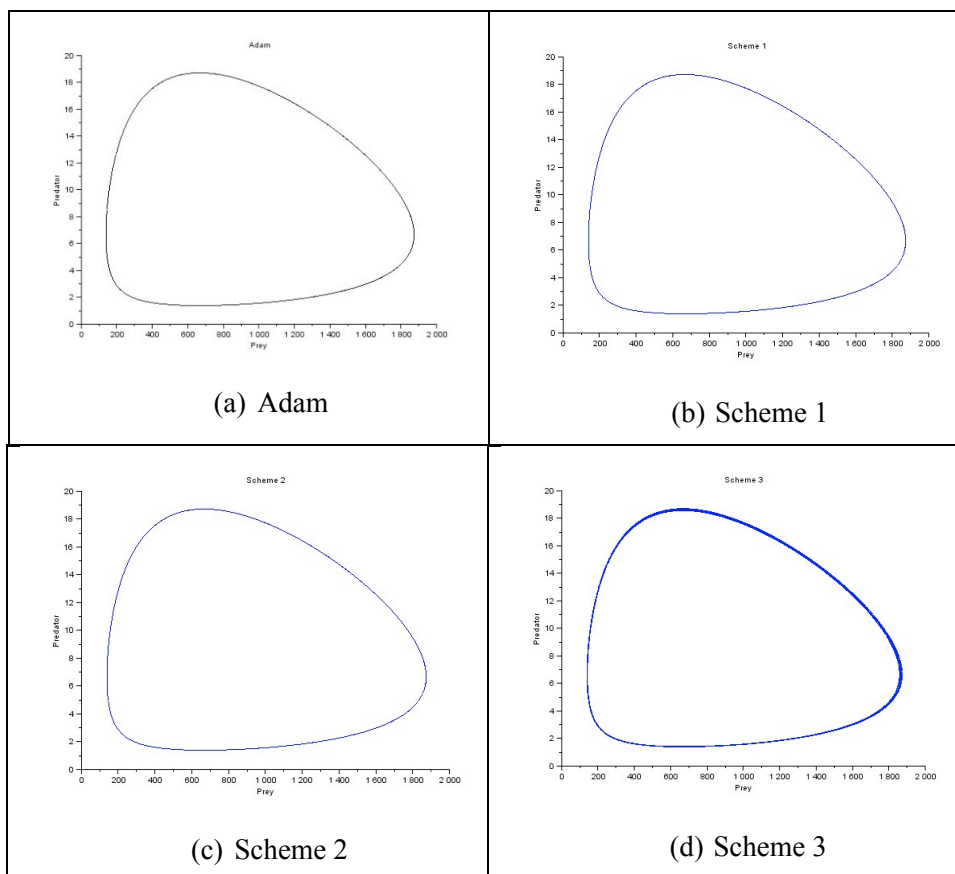


Figure 2. Prey vs predator for parameter set 2

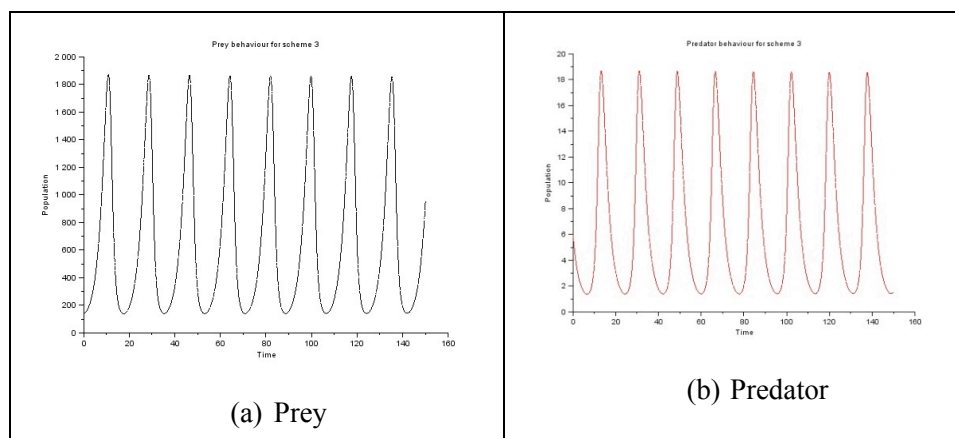


Figure 3. Prey and Predator behaviour for scheme 3 (Parameter Set 2)

Results of scheme 3 is analysed in detail. Extra graph on prey and predator as given in Figure 3 is plotted. From Figure 3, the same number of fluctuations is produced, but the value was quite different.

Figure 4 shows almost similar trend like Figure 2. Scheme 3 showed thicker graph compared with Figure 2.

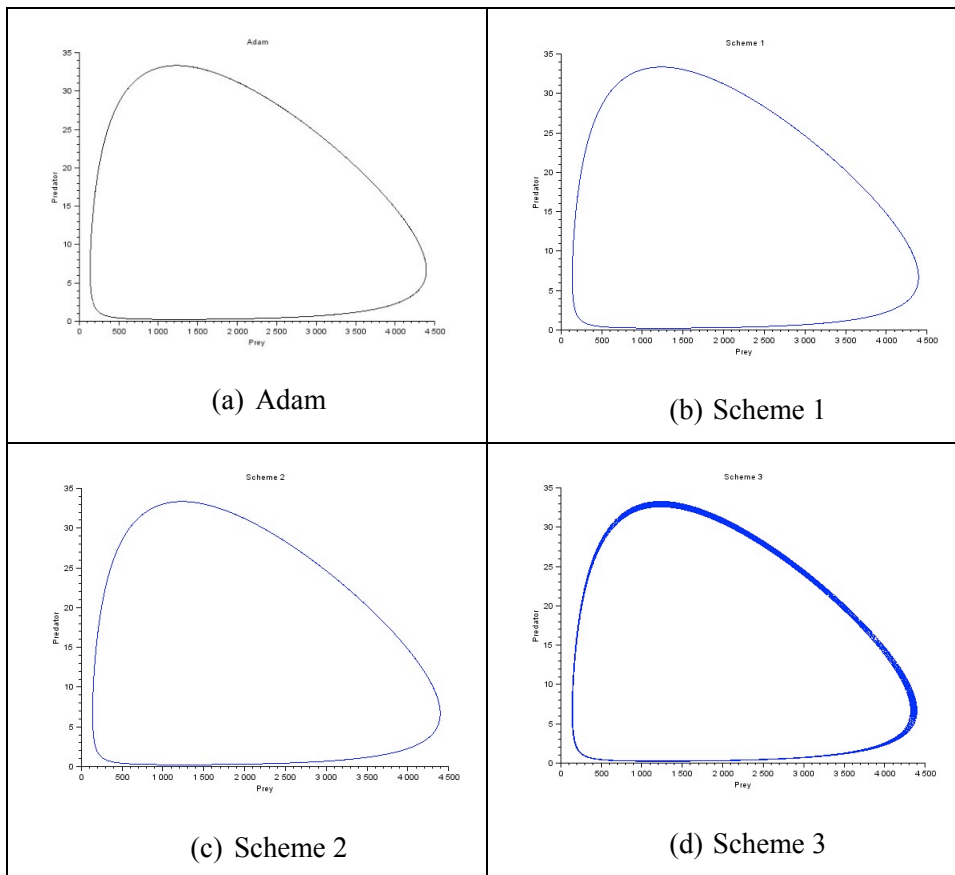


Figure 4. Prey vs predator for parameter set 3

Scheme 3 was analysed using the same approach as before. Extra graph on prey and predator was plotted as shown in Figure 5. Even though it produced the same number of fluctuations, the value was quite different.

Figure 6 shows results simulated for parameter set 4. This figure shows that our new schemes produce smoother results compared with Adam-Bashforth-Moulton method when large mesh size is applied. The new schemes showed more accurate results compared with Adam-Bashforth-Moulton's method since all graphs for schemes 1 and 2 in Figure 6 show less fluctuations. In addition, scheme 3 was able to gather the point of equilibrium.

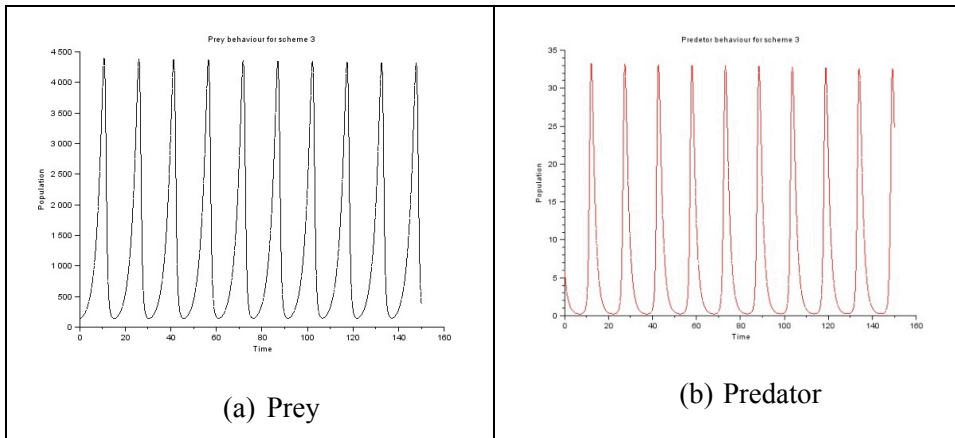


Figure 5. Prey and Predator behaviour for Scheme 3 (Parameter set 3)

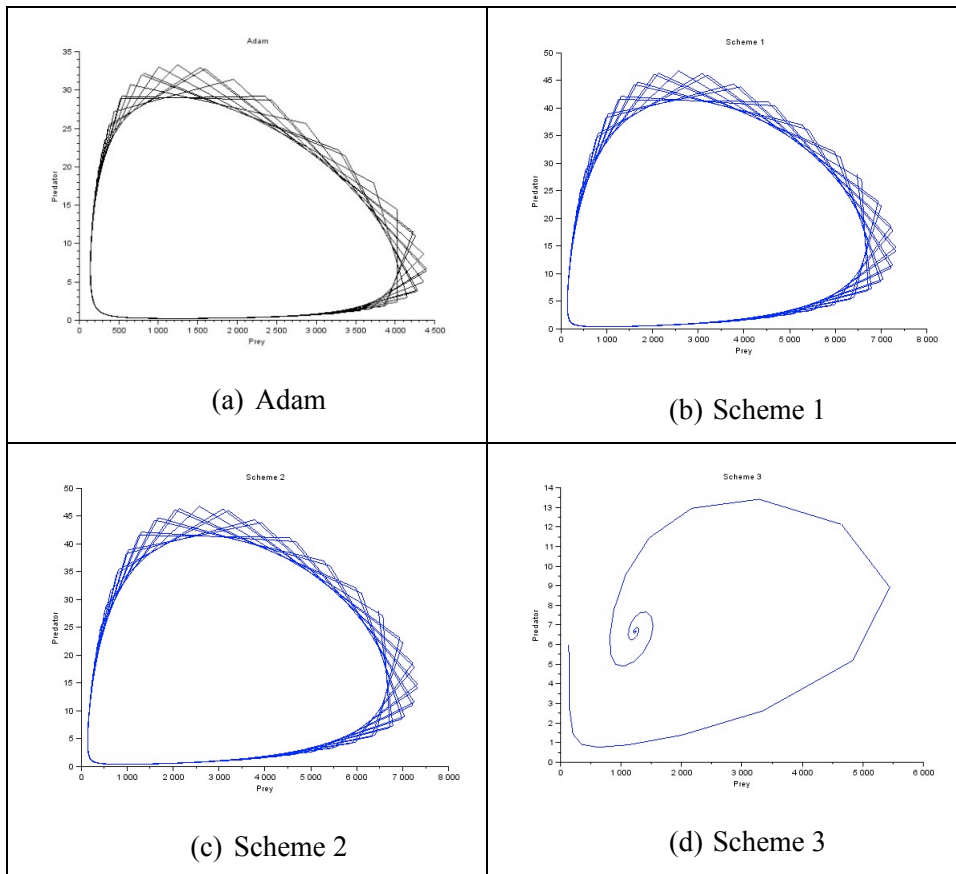


Figure 6. Prey vs predator for parameter set 4

CONCLUSION

In this paper, three new one-step method schemes were proposed and evaluated. In order to analyse the performance of these schemes, they were compared with a high accuracy fourth order predictor-corrector method, Adam-Bashforth-Moulton's method. The method was known for its accuracy; however, it was more complicated than all the new schemes. For small step size, the results were comparable but for larger step size, schemes 1 and 2 were more accurate (than Adam-Bashforth-Moulton's method).

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