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New Hybrid Two-Step Method for Simulating Lotka-Volterra Model

Noor Ashikin Othman* and Mohammad Khatim Hasan

Center for Artificial Intelligence Technology, Faculty of Information Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

ABSTRACT

Simulating Lotka-Volterra model using numerical method require researchers to apply tiny mesh sizes to obtain an accurate result. This approach nevertheless increases the complexity and burden of computer memory and consume long computational time. To overcome these issues, we investigate and construct new two-step solver that could simulate Lotka-Volterra model using bigger mesh size. This paper proposes three new two-step schemes to simulate Lotka-Volterra model. A non-standard approximation scheme with trimean approach was adopted. The nonlinear terms in the model is approximated via trimean approach and differential equation via non-standard denominators. Four sets of parameters were examined to analyse the performance of these new schemes. Results show that these new schemes provide better simulation for large mesh size.

Keywords: Lotka-Volterra, Nonstandard method, Trimean approach, Two-step method

INTRODUCTION

The Lotka-Volterra model was proposed by Alfred James Lotka (1925) and Vito Volterra (1926) in two difference attempt and purposes (Bacaer, 2011). In the model, at least two variables representing the prey and the predators are presented in two differential equations. The prey is assumed to have unlimited source of food, while the only source of food for the predator is the prey. The Lotka-Volterra model has been used in many applications such as transportation (Yuting & Meng, 2011) and security (Yang & Chen, 2015). Jovanović, B., Mostarac, K., Šarac, D., & Rakić, E. (2015) analyse the interaction between corporate and government sectors using Lotka-Volterra, while Yang and Chen (2015) develop a fire security system aimed at awareness of fire security level in university using the same model.

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E-mail addresses:
annisrulwaqi7342@gmail.com (Noor Ashikin Othman),
mkh@ukm.edu.my (Mohammad Khatim Hasan)
*Corresponding Author

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In this paper, three new two-step schemes are developed for simulating the Lotka-Volterra model. The schemes are intended to improve the accuracy of existing numerical approach for big mesh size since numerical method always require the researcher to apply tiny mesh sizes to generate accurate solution. Non-standard approximation and trimean approach are adopted (Mickens, 2003; Yaacob & Hasan, 2015). The trimean (Ji, Wang, Wu, Wu, Xing, & Liang, 2010) concept was used to represent nonlinear terms in Lotka-Volterra model.

MATERIALS AND METHODS

A Lotka-Volterra model with two equations is given as follows.

$$\frac{dx}{dt} = Ax - Bxy, \quad \frac{dy}{dt} = -Cy + Dxy \tag{1}$$

where x and y denote the prey and predator population, $A > 0$ represent prey birth rate, $\boldsymbol{A} > 0$ represent rate of prey consumed by predator, $C > 0$ represent predator death rate and $D > 0$ denotes predator birth rate.

In this section, three new two step method is constructed. The derivation of all three schemes are discussed and approximation $\frac{dx}{dt}$ and $\frac{dy}{dt}$ is done following Mickens (2003), Yaacob and Hasan (2015) and Bhowmik (2009). The derivative is approximated as follows:

$$\frac{dx}{dt} \approx \frac{x_{i+1} - x_i}{\emptyset},$$

$$\frac{dy}{dt} \approx \frac{y_{i+1} - y_i}{\emptyset}$$

Here, we take the denominator function, $\emptyset = \sin(h)$. Scheme 1 is developed by replacing x in eq. (1) at $i = 0$ with $x \rightarrow 2x_0 - x_1$, $xy \rightarrow x_1 y_0$, thus

$$\frac{dx}{dt} = Ax - Bxy \rightarrow \frac{x_1 - x_0}{\emptyset} = A(2x_0 - x_1) - B(x_1 y_0)$$

$$x_1 - x_0 = A\emptyset(2x_0 - x_1) - B\emptyset(x_1 y_0)$$

$$x_1 - x_0 = (2A\emptyset x_0 - A\emptyset x_1) - (B\emptyset x_1 y_0)$$

$$x_1 = \frac{2A\emptyset x_0 + x_0}{(1 + A\emptyset + B\emptyset y_0)}.$$

The trimean approach is applied at $i = 1$, thus

$$\frac{x_{i+1} - x_i}{\emptyset} = A\left(\frac{x_{i-1} + 2x_i + x_{i+1}}{4}\right) - Bx_{i+1}y_i$$

$$x_{i+1} - x_i = 0.25A\emptyset(x_{i-1} + 2x_i + x_{i+1}) - B\emptyset(x_{i+1}y_i) \tag{2}$$

$$x_{i+1} = \frac{0.25A\emptyset x_{i-1} + 0.5A\emptyset x_i + x_i}{(1 - 0.25A\emptyset + B\emptyset y_i)}$$

and by taking $y \to -y_i + 2y_{i+1}, xy \to 2x_{i+1}y_i - x_iy_{i+1}$, thus

$$\frac{dy}{dt} = -Cy + Dxy \to \frac{y_{i+1} - y_i}{\emptyset} - C(-y_i + 2y_{i+1}) + D(2x_{i+1}y_i - x_iy_{i+1})$$

at $i = 0$

$$y_1 = \frac{C\emptyset y_0 + 2D\emptyset x_1 y_0 + y_0}{(1 + 2C\emptyset + D\emptyset x_0)}.$$

The trimean approach was applied at $i = 1$, thus

$$\frac{y_{i+1} - y_i}{\emptyset} = -C\left(\frac{y_{i-1} + 2y_i + y_{i+1}}{4}\right) + D(2x_{i+1}y_i - x_iy_{i+1})$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1}y_i + y_i}{(1 + 0.25C\emptyset + D\emptyset x_i)} \tag{3}$$

Eq. (2) and (3) represent scheme 1.

We develop scheme 2 by replacing $xy \to 2x_{i+1}y_i - x_iy_{i+1}$, while the other term remains the same as in scheme 1, thus at $i = 0$,

$$\frac{y_1 - y_0}{\emptyset} = -C(y_1) + D(2x_1y_0 - x_0y_1),$$

$$y_1 = \frac{2D\emptyset x_1 y_0 + y_0}{(1 + C\emptyset + D\emptyset x_0)}$$

The trimean approach was applied at $i = 1$ thus

$$\frac{y_{i+1} - y_i}{\emptyset} = -C\left(\frac{y_{i-1} + 2y_i + y_{i+1}}{4}\right) + D(2x_{i+1}y_i - x_iy_{i+1})$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1}y_i + y_i}{(1 + 0.25C\emptyset + D\emptyset x_i)} \tag{4}$$

We develop scheme 3 by replacing $xy \to 2x_{i+1}y_i - x_{i+1}y_{i+1}$, while other terms remain the same as in scheme 1, thus at $i = 0$

$$\frac{y_1 - y_0}{\emptyset} = -C(y_1) + D(2x_1y_0 - x_1y_1),$$

$$y_1 = \frac{2D\emptyset x_1 y_0 + y_0}{(1 + C\emptyset + D\emptyset x_1)}$$

The trimean approach was applied at $i = 1$, thus

$$\frac{y_{i+1} - y_i}{\emptyset} = -C\left(\frac{y_{i-1} + 2y_i + y_{i+1}}{4}\right) + D(2x_{i+1}y_i - x_{i+1}y_{i+1})$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1}y_i + y_i}{(1 + 0.25C\emptyset + D\emptyset x_{i+1})} \quad (5)$$

The algorithm for scheme 1 was constructed by using approximate Eq. (2) and (3) and shown in Algorithm 1, while algorithm for scheme 2 was constructed using approximate Eq. (2) and (4) and shown in Algorithm 2, and for scheme 3 using (2) and (5) and shown in Algorithm 3. We conduct an experiment with four sets of parameters (Prian, 2013). The parameters are

a. $A = 0.4$ $B = 0.11$ $C = 0.12$ $D = 0.0032, h = 0.001, x_0 = 140,$
   $y_0 = 6\ (A > C)$

b. $A = 0.4, B = 0.11, C = 0.4, D = 0.0032, h = 0.001, x_0 = 140,$
   $y_0 = 6\ (A = C)$

c. $A = 0.4, B = 0.11, C = 0.74, D = 0.0032, h = 0.001, x_0 = 140,$
   $y_0 = 6\ (A < C)$

d. $A = 0.4, B = 0.11, C = 0.74, D = 0.0032, h = 1.0, x_0 = 140,$
   $y_0 = 6\ (h = 1.0)$

Algorithm 1: Algorithm for scheme 1

Set

$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$

Calculate x and y using

$$x_2 = \frac{2A\emptyset x_1 + x_1}{(1 + A\emptyset + B\emptyset y_1)}$$

$$y_2 = \frac{C\emptyset y_1 + 2D\emptyset x_2 y_1 + y_1}{(1 + 2C\emptyset + D\emptyset x_1)}$$

Then calculate x and y $(i = 2, \dots, n-1)$ using

$$x_{i+1} = \frac{0.25A\emptyset x_{i-1} + 0.5A\emptyset x_i + x_i}{(1 - 0.25A\emptyset + B\emptyset y_i)}$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1} y_i}{(1 + 0.25C\emptyset + D\emptyset x_i)}$$

Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. Predator

Algorithm 2: Algorithm for scheme 2

Set

$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$

Calculate x and y using

$$x_2 = \frac{2A\emptyset x_1 + x_1}{(1 + A\emptyset + B\emptyset y_1)}$$

$$y_2 = \frac{2D\emptyset x_2 y_1 + y_1}{(1 + C\emptyset + D\emptyset x_1)}$$

Then calculate x and y $(i = 2, \dots, n-1)$ using

$$x_{i+1} = \frac{0.25A\emptyset x_{i-1} + 0.5A\emptyset x_i + x_i}{(1 - 0.25A\emptyset + B\emptyset y_i)}$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1} y_i}{(1 + 0.25C\emptyset + D\emptyset x_i)}$$

Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. Predator

Algorithm 3: Algorithm for scheme 3

Set

$t_0, x_0, y_0, A, B, C, D, w, t\ range, y\ range, h, P$

Calculate x and y using

$$x_2 = \frac{2A\emptyset x_1 + x_1}{(1 + A\emptyset + B\emptyset y_1)}$$

$$y_2 = \frac{2D\emptyset x_2 y_1 + y_1}{(1 + C\emptyset + D\emptyset x_2)}$$

Then calculate x and y $(i = 2, \dots, n-1)$ using

$$x_{i+1} = \frac{0.25xA\emptyset_{i-1} + 0.5A\emptyset x_i + x_i}{(1 - 0.25A\emptyset + B\emptyset y_i)}$$

$$y_{i+1} = \frac{-0.25C\emptyset y_{i-1} - 0.5C\emptyset y_i + 2D\emptyset x_{i+1} y_i}{(1 + 0.25C\emptyset + D\emptyset x_{i+1})}$$

Output: $x_{min}, x_{max}, y_{min}$ and y_{max} graph for prey vs. Predator

Scilab was used to code the algorithms in Algorithm 1-3. We compare our simulated results with that generated using Adam-Bashforth-Moultan method (Hasan, Karim, & Sulaiman, 2015), which is $O(h^4)$ for accuracy.

RESULTS AND DISCUSSIONS

Graphs plotted by our code are given in Figures 1-4. Figures 1-3 show that all method produces exactly almost similar result. These show the new two-step scheme simulates results which are comparable with that obtained using Adam-Bashforth-Moultan method. However, scheme 1 and 2 produce thicker result compared with others. Even though it produces the same number of fluctuations, the values are quite different.

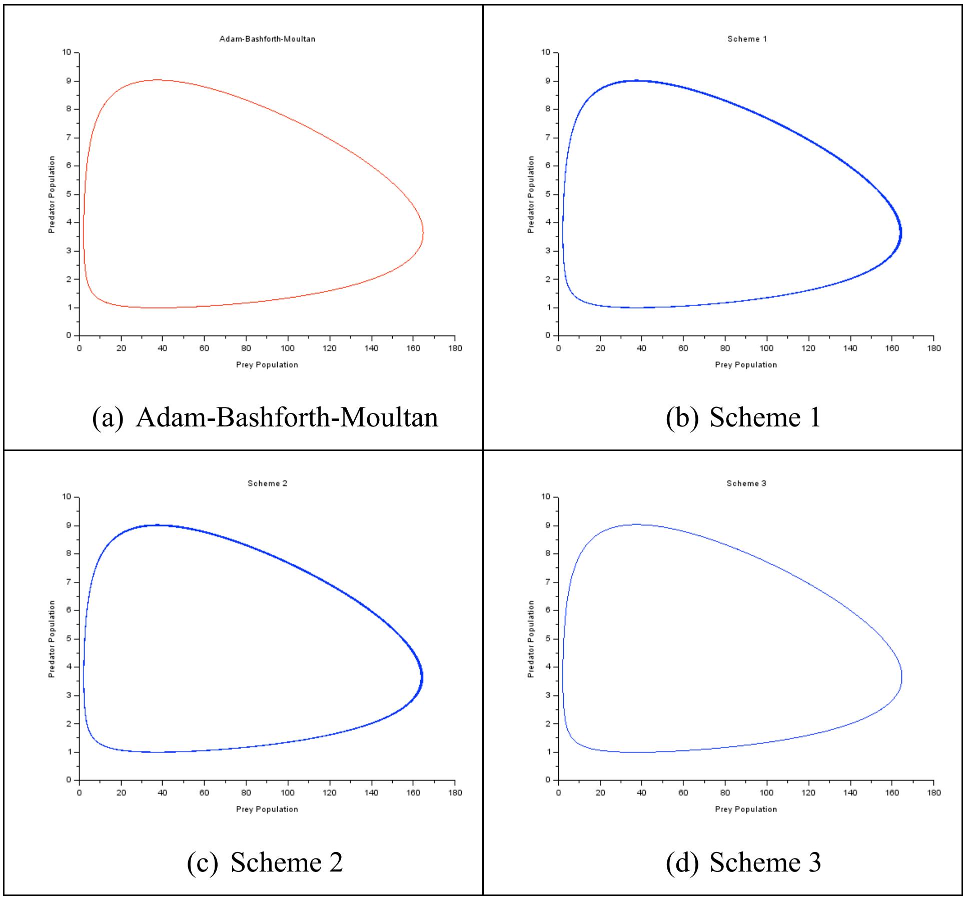
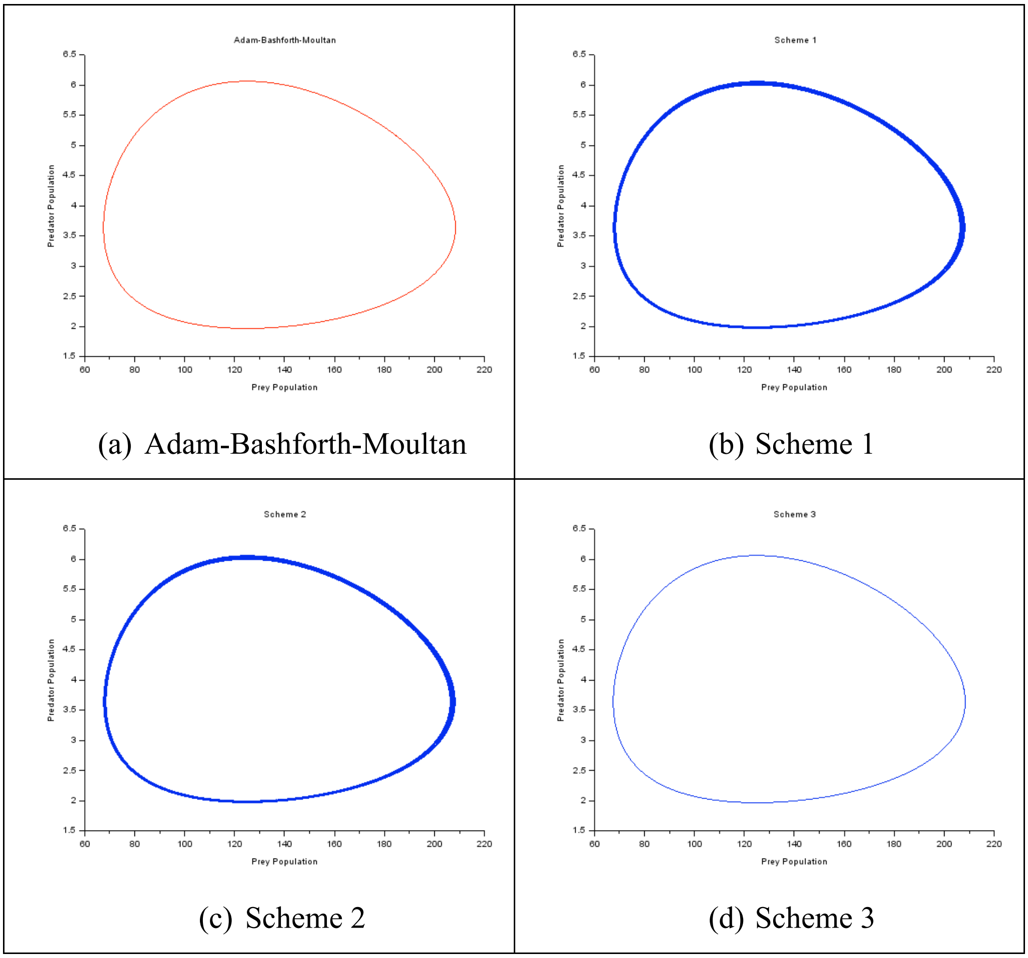


Figure 1. Prey vs predator for parameter set 1

Figure 4 shows results simulated for parameter set 4. It is clear schemes 1 and 2 produce the same output, while Adam-Bashforth-Moultan and scheme 3 produce different behaviours. Using larger h does not change the interaction behaviour of prey and predator simulated using Adam-Bashforth-Moultan method. While using larger h does affect the behaviour of our schemes. Schemes 1 and 2 end at its exact equilibrium point, while scheme 3 produces a very interesting behaviour. The exact equilibrium point was calculated assuming that there were no further changes in both predator and prey in time; i.e.; by taking $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in Eq. (1) to be zero.



(a) Adam-Bashforth-Moultan (b) Scheme 1

(c) Scheme 2 (d) Scheme 3

Figure 2. Prey vs predator for parameter set 2

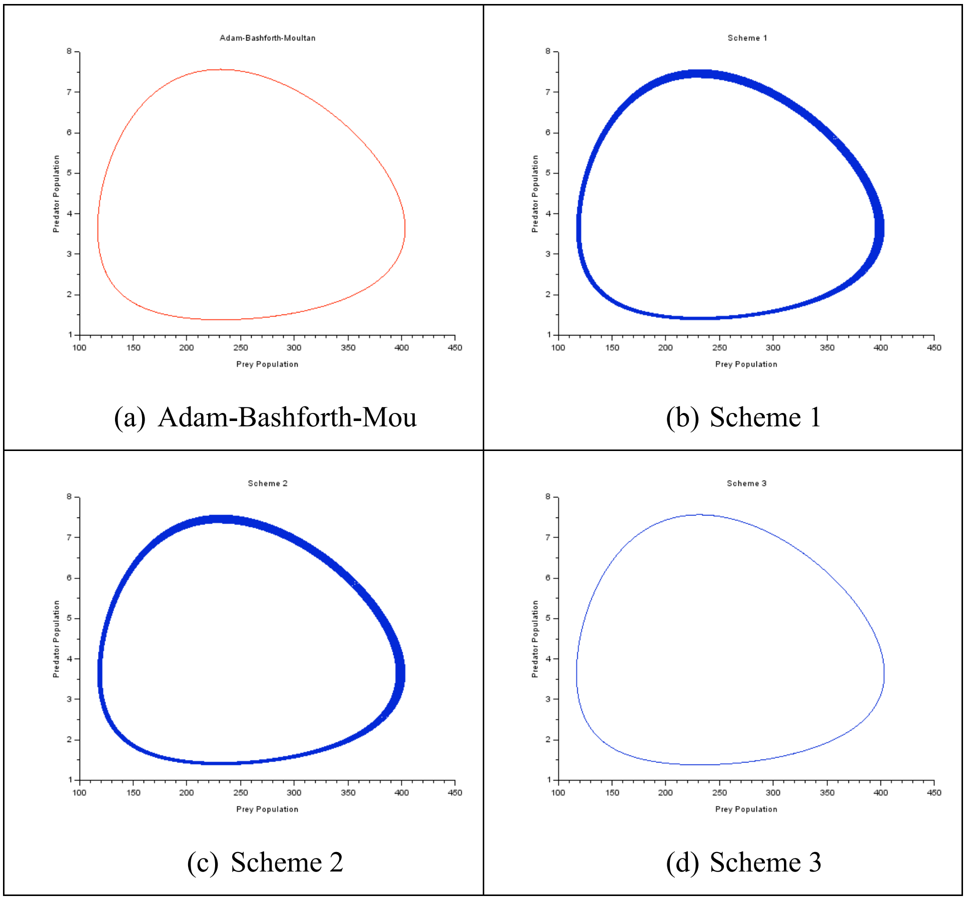


Figure 3. Prey vs predator for parameter set 3

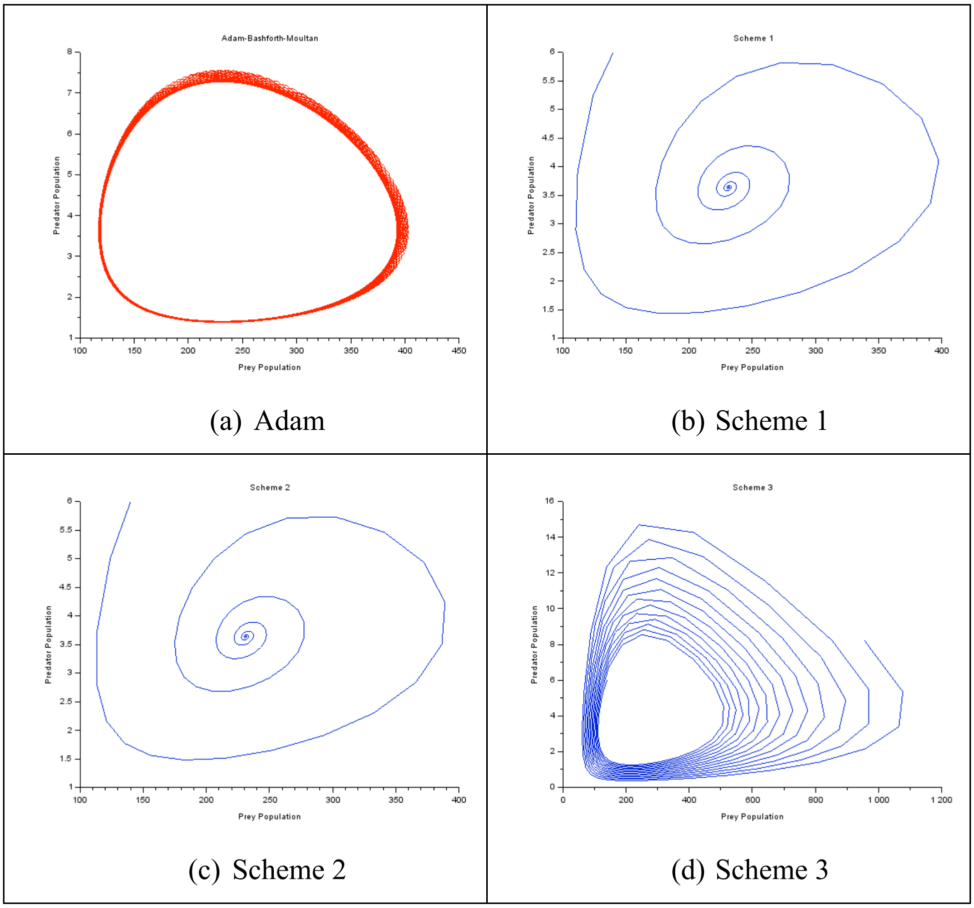


Figure 4. Prey vs predator for parameter set 4

CONCLUSION

This paper showed three new two-step schemes. In order to analyse the performance of these schemes, they are compared with a high accuracy fourth order predictor-corrector method, Adam-Bashforth-Moultan method. The latter method is known for its accuracy; however, it is more complicated than all study's new schemes. For small step size, the present results are comparable with Adam's. But for larger step size, scheme 1 and 2 is able to gather the point of equilibrium, while scheme 3 produces extremely interesting behaviour and need further analysis.

In future research, the present authors will apply this algorithm to solve higher order system of ordinary differential equations and other predator-prey models such as Rosenzweig-MacArthur and Beddington-DeAngelis.

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