

SCIENCE & TECHNOLOGY

Journal homepage: http://www.pertanika.upm.edu.my/

Application of the First Order of Markov Chain Model in Describing the PM10 Occurrences in Shah Alam and Jerantut, Malaysia

Mohamad, N. S.1*, Deni, S. M.1 and Ul-Saufie, A. Z.2

¹Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 UiTM, Shah Alam, Malaysia

²Department of Computer and Mathematical Sciences, Universiti Teknologi MARA, 13500 UiTM, Permatang Pauh, Pulau Pinang, Malaysia

ABSTRACT

 PM_{10} has been identified as being a common problem in Malaysia and many other countries all over the world. A Markov chain probability model is found to fit the average daily PM_{10} concentrations data of urban station (Shah Alam) and background area station (Jerantut) in Malaysia. This study aims to identify the occurrence of polluted and non-polluted days affected by PM_{10} concentrations based on data for 12 years' period (2002-2013). The first order transition probability matrix of a Markov chain model and a two-state Markov chain, which are polluted days (1) and non-polluted days (0), were used for this purpose. The threshold value used in this study is referring to WHO 2006 guidelines ($50\mu gm^{-3}$). Results of the analysis shows that there is a high probability that the next day event depends on what has happened on the previous day. The recurrence of the polluted day for Shah Alam is 4-5 days, while 2-3 days for Jerantut. By fitting the first order of Markov chain model, the results show that the higher order of Markov chain model is needed in order to get the best fitted distribution of polluted events at these two monitoring stations. Thus, the prediction of PM_{10} concentrations event can be made by considering the conditions of the previous day event.

Keywords: Markov chain model, PM₁₀ concentrations, polluted days, non-polluted days, occurrence

ARTICLE INFO

Article history:

Received: 03 March 2017 Accepted: 28 September 2017

E-mail addresses:
norsalwani630217@gmail.com (Mohamad, N. S.),
sayang@tmsk.uitm.edu.my (Deni, S. M.),
ahmadzia101@ppinang.uitm.edu.my (Ul-Saufie, A. Z.)
*Corresponding Author

ISSN: 0128-7680 © 2018 Universiti Putra Malaysia Press.

INTRODUCTION

Particles or particulate matter is a type of air pollution. PM_{10} , which is a particulate matter with aerodynamic diameter less than 10 micrometre, has been identified as being a common problem in Malaysia and some other countries. Furthermore, PM_{10} is also mainly considered in Malaysia as this pollutant gives

greatest concern to public health, apart from being the main air pollutant that has exist since the haze event in 1980 (Awang et al., 2000). Cardiovascular diseases, respiratory problems and cancer are some examples of health problems caused by this air pollutant (EPA, 2001). PM₁₀ is emitted by both anthropogenic and natural sources. There are five major PM₁₀ emissions in Malaysia, which are motor vehicle exhausts, industrial sources, open burning, heat and power plants (DoS, 2013). However, the main causes of this pollutant in Malaysia are industrial activities and heavy traffic (Ul-Saufie et al., 2012). Many efforts have been done in order to monitor the effects of this pollutant; these include the regression model that is used to predict PM₁₀ concentrations (Paschalidou et al., 2011; Chaloulakou et al., 2003; Pires et al., 2008). However, the study of the sequence of polluted and non-polluted days using Markov chain model still receives less attention.

Markov chain model was introduced at the beginning of the 20th century and it has been used in many disciplines including geography, meteorology, biology, physics, behaviour, social sciences, chemistry and others (Berchtold & Raftery, 2002). Many researchers have used this method to model the daily occurrences of rainfall and have successfully used it especially for the first order (Gabriel & Neumann, 1962; Mangaraj et al., 2013; Basu, 1971). The first order Markov chain is simple and it does not require a lot of computational efforts. The study of Berchtold and Raftery (2002) successfully used Markov chain model in describing the occurrence of daily rainfall in Tel Aviv for the mid-winter season. Besides that, other researchers such as Haan et al., (1976), Jimoh and Webster, (1996), Rahimi et al. (2011) and Chin (1977) also used Markov chain model in their fields of study. The order of the chain gives the number of time steps in the past influencing the probability distribution of the present state, which could be greater than one. Several authors have found that the order more than one gave the optimum result for certain conditions. For example, Deni et al. (2009) found that the first order of Markov chain model is the optimum order for the rainfall occurrence during the northwestern and eastern regions of peninsula, while a higher order is fit for the rainfall occurrence during the northeast monsoon season. However, this study only considers the first order of Markov chain model because of its simplicity.

The main purpose of this study is to determine the occurrence of polluted and non-polluted days affected by PM₁₀ concentrations at an urban area (Shah Alam) and a background area (Jerantut) in Malaysia. This kind of study still receives less attention from other researchers. This study was conducted as follows. After a brief description of the PM₁₀ concentrations at both stations in the data and methodology section, the explanation on how to determine the polluted day and non-polluted day and the transition probability are also discussed. Testing for the Markov chain property is also discussed in this section to determine whether or not this study possesses the first order of Markov property. This is followed by the persistency of the polluted (non-polluted) events. Results of the analysis and conclusion of this study are discussed in the results and discussion, and conclusion, respectively.

DATA AND METHODOLOGY

Study Area and Data Preparation

Shah Alam is the state capital of Selangor, Malaysia (Figure 1). It is located about 25 kilometres west of the country's capital, Kuala Lumpur. Shah Alam monitoring station is located at Sekolah Kebangsaan Raja Muda, Shah Alam. This station was chosen based on its land use background (urban area) as the absence of PM₁₀ concentrations is higher at this area because of the daily activities and heavy traffic emissions. In Jerantut, the background station is located at the Meteorology Monitoring Station, Batu Embun, Jerantut. This monitoring station was selected as the background area because it is situated at the rural area and still has low potential of recording air pollution in Malaysia (Latif et al., 2014). The latitude and longitude for both stations are shown in Table 1.

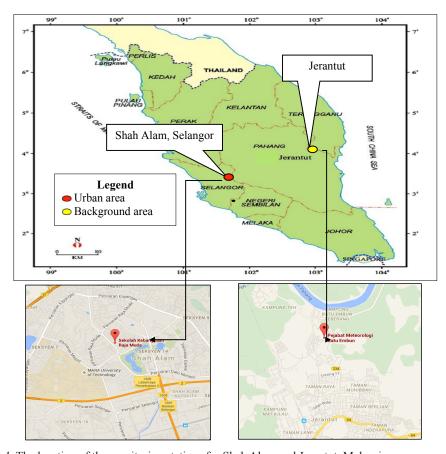


Figure 1. The location of the monitoring stations for Shah Alam and Jerantut, Malaysia

Data for twelve years (2002-2013) of the PM_{10} concentration monitoring, provided by the Department of Environment (DoE) Malaysia, are used in this study. In this study, the hourly data of PM_{10} concentrations were transformed into daily average data. However, the PM_{10} concentrations data were incomplete due to missing values as they were obtained using an automated machine. Thus, according to Junninen et al. (2004), the missing value for the air pollution monitoring record is in the category of missing at random and as the data used in this study contained small number percentage of missing value (which is < 1%) (Table I), the missing value is completely ignored. Little and Rubin (2002) stated that the low percentage of missing values are use the easiest and most common approach method to deal with these missing values, which is to completely ignore the missing values (delete all the missing values) and continue with the complete datasets.

Table 1
Basic descriptions of the monitoring stations and descriptive statistics

| Station | Shah Alam | Jerantut | |
|-------------------------------|-----------|----------|--|
| Latitude (°N) | 3.08 | 3.95 | |
| Longitude (°E) | 101.51 | 102.36 | |
| Valid Data | 4369.00 | 4342.00 | |
| % missing | 0.32 | 0.94 | |
| Mean (μg/m³) | 56.28 | 38.63 | |
| Median (µg/m³) | 51.57 | 36.22 | |
| Standard Deviation (µg/m³) | 26.15 | 15.27 | |
| Minimum value (µg/m³) | 14.00 | 10.00 | |
| Maximum value ($\mu g/m^3$) | 587.00 | 211.00 | |

Methodology

The PM_{10} concentrations were divided into two states: polluted day and non-polluted day. A polluted day can be defined as a day when the PM_{10} concentrations exceed or is equal to a certain threshold value, while the day is considered as a non-polluted day when the PM_{10} concentrations are lesser than the threshold value. According to DoE (2013), the current guidelines used in Malaysia for all the parameters, including PM_{10} concentration, are less relevant and outdated. Thus, this study considers the threshold value based on the World Health Organization (WHO), which is the PM_{10} concentrations should not exceed 50 $\mu g/m^3$ for 24 hours as one of the issues discussed and updated in Malaysia Ambient Air Quality Standard 2013 as the new standard in the line with WHO 2006 guidelines (see Table 2). Table 2 was adopted from ADB and CAI-Asia (2006).

Table 2
The Malaysia and WHO 2006 ambient air quality guidelines

| Pollutant | Averaging Time | Malaysian Ambient Air Quality Guidelines | WHO (2006) |
|-----------|----------------|---|------------|
| PM_{10} | 24 hrs | 150 | 50 |
| | 1 year | 50 | 20 |

The Markov Chain is intended to be a simple model by requiring only two parameters. The assumptions in this model are that the probability of a polluted (or non-polluted) day depends only on whether it has polluted (or not) on the previous day; and the probability of polluted (or non-polluted) is assumed to be independent of the preceding days.

These probabilities are also known as transitional probabilities, denoted by P_{11} and P_{00} for the sequence of two-polluted days and two-consecutive non-polluted days, respectively. The estimation of P_{11} and P_{00} is directly counting methods from the available daily PM_{10} concentrations record. The formula for first order of Markov is given in equation 1 and 2.

$$P_{11} = P(X_t = 1 \mid X_{t-1} = 1)$$
 (1)

$$P_{01} = P(X_t = 0 \mid X_{t-1} = 1)$$
 (2)

The transition probability matrix P, which describes the 2-state Markov chain model, is as in Equation 3 below.

$$P = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix} \tag{3}$$

The polluted days are denoted by 1 and non-polluted days are denoted by 0. Each of the elements in this matrix is defined as:

P₀₀: Probability of a non-polluted day given that the previous day was also a non-polluted day

 P_{01} : Probability of a polluted day given that the previous day was a non-polluted day

P₁₀: Probability of a non-polluted day given that the previous day was a polluted day

P₁₁: Probability of a polluted day given that the previous day was also a polluted day

The calculation of each of the elements in the matrix is as follows:

$$P_{01} = \frac{n_{01}}{n_{01} + n_{00}} \rightarrow P_{00} = 1 - P_{01} \tag{4}$$

$$P_{10} = \frac{n_{10}}{n_{10} + n_{11}} \to P_{11} = 1 - P_{10}$$
(5)

n denotes the total number of days for a given state.

The Markov chain properties of the daily occurrence can be tested statistically by checking whether or not the successive events are independent of each other. If the successive events are dependent, the events can form the first-order Markov chain model. The hypotheses are as follows:

H₀: The successive events are independent

H₁: The successive events are dependent

According to Moon et al. (1994), if the successive events are independent, then the statistics, α , is defined by

$$\alpha = 2\sum_{i,j}^{m} n_{ij} \ln(P_{ij}/P_j) \tag{6}$$

Where equation 6 is distributed asymptomatically as χ^2 with $(m-1)^2$ degree of freedom. Meanwhile, m is the total number of state (in this case: m=2) and the marginal probabilities for j^{th} column of the transition probabilities matrix is presented by P_i .

Besides that, P_{11} and P_{00} can be used to determine the average lengths of the polluted and non-polluted spell, as follows:

The average length of polluted spells is

$$\frac{1}{1 - P_{11}} \tag{7}$$

whereas the average length of non-polluted spells is

$$\frac{1}{1 - P_{00}} \tag{8}$$

Bessons's coefficient of persistence is used to determine whether persistency is commonly used in meteorology. The formula has the following form:

$$R_B = \frac{1 - P_1}{1 - P_{11}} - 1 \tag{9}$$

where P_1 is the probability of a polluted day and P_{11} is the probability of a polluted day, given that the previous day was also a polluted day. The positive value of R_B indicates that the occurrence of a polluted or non-polluted event followed by an immediate preceding event. If the value is zero, then it indicates no persistence for a similar event. Many other researchers such as Deni et al. (2009) and Dahale et al. (1994) have used this particular method.

In fitting the first order of the Markov chain model, the expected distribution closer to the observed distribution of polluted events was analysed. The purpose of fitting this Markov chain model was to check whether the first order obtained was the appropriate fitted order to describe the observed distribution of polluted events for the data set at these two monitoring stations. The first order of the Markov chain model considers only the condition of the state, either polluted or non-polluted, for one preceding day only. Equation 10 shows the probability of the duration of n days for the polluted events.

$$P_{(n)} = P_{11}^{n-1} \times P_{10} \tag{10}$$

The chi-square goodness of fit test was employed to compare the observed and expected distributions of polluted days. The calculation of the chi-square test statistics is as follows (Skuriat-Olechnowska, 2005):

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \tag{11}$$

Where O_i = observed data

 E_i = Expected data

This statistics follows the chi-square distribution, with a number of degree of freedom (d=v-1), where v is the length of polluted days. All the expected frequencies must be at least five days and the hypothesis is that the data follows the observed distribution when $\chi^2 < \chi^2_{1-\alpha,v-1}$ is at 5% significant level.

RESULTS AND DISCUSSION

Over the study period of 12 years, Shah Alam recorded the maximum value of PM_{10} concentration at 587 µg/m³, while the maximum value of PM_{10} concentrations for Jerantut was 211 µg/m³ (see Table 1). The absence of this pollutant at both stations is over the threshold value of PM_{10} concentrations for certain days. Besides that, the average daily PM_{10} concentrations for Shah Alam and Jerantut shows that for both stations, the PM_{10} concentrations are higher between day 177 (June) up to day 221 (August), as shown in Figure 2. This result indicates that the present of this pollutant is higher during the South west monsoon (June to September) or dry weather condition (Yusof et al., 2011). This study also shows that Shah Alam recorded the highest number of polluted days (2396 days), whereas Jerantut recorded the highest number of non-polluted days (3552 days).

The frequencies of the first order transitions by considering today's polluted (1) and non-polluted (0) day, followed by yesterday's events (polluted or non-polluted day) for both the stations, are shown in Table 3. Shah Alam recorded the highest frequency of polluted days given that the previous day was also a polluted day, while Jerantut recorded the highest frequency of a non-polluted day, given that the previous day was also a non-polluted day. These results show that Jerantut still has low potential of recording air pollution in Malaysia.

Table 3 also shows results of the transition probabilities for a first order Markov chain obtained directly by using the transition counts using the formula in equation 4 and equation 5 for both stations. Shah Alam and Jerantut recorded a high probability of the next day being polluted when the previous day was polluted (P_{11}) and also recorded a low probability for the next day being not polluted when the previous day was polluted (P_{10}). This finding can help

responsible authorities to take action by providing early warning to the respective population when the previous day is a polluted day and also made prediction of PM_{10} event by referring to the condition of the previous day event.

Table 3

Tally matrix and transition probabilities of the occurrence of polluted and non-polluted days for 12 years at both stations

| Station | Tally Matrix | | Transition probabilities | | |
|-----------|--------------|------|--------------------------|--------|--|
| Shah Alam | 1472 | 501 | 0.7461 | 0.2539 | |
| | 502 | 1894 | 0.2095 | 0.7905 | |
| Jerantut | 3271 | 281 | 0.9209 | 0.0791 | |
| | 281 | 513 | 0.3539 | 0.6461 | |

The successive state of daily PM_{10} concentrations was checked by computing a statistic of independence, α , and the results are shown in Table 4. The value of α at both the stations are larger than the χ^2 value of 3.841 at the 5 per cent level, with one degree of freedom. Thus, H_0 is rejected and the successive transitions are dependent. Hence, this study concludes that the transitions of daily PM_{10} concentrations occurrence possesses the first-order Markov chain model property, where the successive events are dependent on each other. The analysis of the mean length of polluted and non-polluted days, based on equation 7 and equation 8 for Shah Alam, indicates that on average, 4-5 days for the recurrence of the polluted days, while this is about 3-4 days for the recurrence of the non-polluted day. On average, Jerantut showed 2-3 days for the recurrence of the polluted day and 12-13 days for the recurrence of non-polluted days. Thus, these reflect the fact that Jerantut is relatively a less polluted area compared to Shah Alam.

Table 4
The mean length of polluted and non-polluted days for both stations

| Station | | Mean | | Besson's |
|-----------|---------------|-------------------|--------|-------------------------------|
| | Polluted days | Non-polluted days | | coefficient (R _B) |
| Shah Alam | 4.77 | 3.94 | 660.22 | 1.61 |
| Jerantut | 2.83 | 12.64 | 567.92 | 1.31 |

Analysis of the persistency of the polluted (non-polluted) events was done by applying the Besson's coefficient of persistence (R_B) as in equation 9. Table 4 shows results of the coefficient and for both stations, the value of R_B was found to be positive, indicating that the occurrence of a polluted (or non-polluted) event was followed by an immediate preceding event. The result shows that the land used background has a strong influence on persistency since the persistent polluted events in Shah Alam were relatively much longer compared to Jerantut.

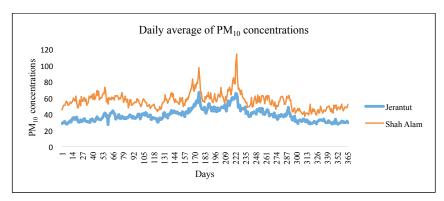


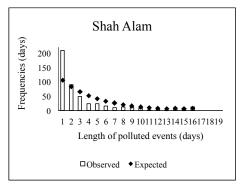
Figure 2. The average daily PM₁₀ concentrations in Shah Alam and Jerantut, Malaysia, for a period of twelve years (2002-2013)

Results of the chi-square test at 5% level of significant shown in Table 5 show that the first order of Markov chain model does not a good fitting on the PM₁₀ concentrations data at the two monitoring stations. The value of chi-square at both monitoring stations (χ^2) have bigger value than $\chi^2_{0.95,\nu-1}$ and the hypothesis that the data follow the observed distribution cannot be accepted at 5% significant level.

Table 5
The observed and expected frequency distributions computed with the first order of Markov chain model in Shah Alam and Jerantut monitoring stations

| Length of polluted day | Shah Alam | | Jerantut | |
|--|----------------|----------------|----------------|---------------|
| | Observed (day) | Expected (day) | Observed (day) | Expected(day) |
| 1 | 209 | 105 | 131 | 99 |
| 2 | 90 | 83 | 52 | 64 |
| 3 | 49 | 66 | 44 | 41 |
| 4 | 24 | 52 | 15 | 27 |
| 5 | 24 | 41 | 8 | 17 |
| 6 | 16 | 32 | 7 | 11 |
| 7 | 10 | 26 | 3 | 7 |
| 8 | 13 | 20 | 4 | 8 |
| 9 | 12 | 16 | 16 | 6 |
| 10 | 8 | 13 | - | - |
| 11 | 2 | 10 | - | - |
| 12 | 5 | 8 | - | - |
| 13 | 1 | 6 | - | - |
| 14 | 6 | 9 | - | - |
| 15 | 6 | 6 | - | - |
| 16 | 13 | 8 | - | - |
| Total | 488 | 501 | 280 | 283 |
| Degree of freedom χ ² p-value | 15.00 | | 8.00 | |
| | 173.30 | | 45.32 | |
| | 2.20E-16 | | 3.20E-07 | |

Figure 3 shows the observed and expected frequencies of polluted events for the first order of Markov chain model for the Shah Alam and Jerantut monitoring stations. From the figures, it can be seen that the expected distribution of polluted events obtained does not closely describe the observed distribution. Thus, it can concluded that there is a need to use the higher order Markov chain model in assessing the best or appropriate fitting in describing the distribution of the polluted events.



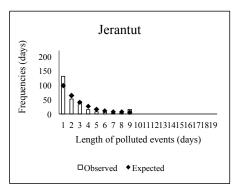


Figure 3. The observed (line) and expected (dot) frequencies of polluted event of the first order for both stations

CONCLUSION

The results obtained from the analysis can provide a better understanding of PM₁₀ occurrences and the pattern at these two monitoring stations. In this study, the first order of Markov chain model was applied and the threshold value used was referring to WHO 2006 guidelines (50µgm⁻³). The results of the analysis show the high probability that the next day event depends on what has happened on the previous day. Besides that, the analysis of persistency showed that for both stations, the occurrence of polluted (non-polluted) event, followed by an immediate preceding event and the recurrence of polluted day in Shah Alam (4-5 days), is longer compared to Jerantut (2-3 days). By fitting the first order of Markov chain model, the results showed that the higher order of Markov chain model is needed in order to get the best fitted distribution of polluted events at these two monitoring stations. This study can help the responsible authorities to provide early warning to the public so that they can avoid from being exposed to air pollutant when the previous day is a polluted day. Further analysis could be done by considering other monitoring stations in Malaysia and different threshold value of PM_{10} concentrations in order to get more valuable information for the purpose of prediction. Besides that, the higher order (order more than one) could be carried out to find the optimum order that best describes the occurrence of PM₁₀ concentrations in Malaysia and thus, a better prediction can be made.

ACKNOWLEDGEMENT

This study was funded by Universiti Teknologi MARA (UiTM), under the Grant 600-RMI/IRAGS 5/3 (36/2015). Special recognition goes to the Department of Environment (DoE) and Alam Sekitar Malaysia Sdn. Bhd. (ASMA) for providing the air quality data for this study. Special appreciation also goes to Universiti Teknologi MARA (UiTM).

REFERENCES

- ADB & CAI-Asia. (2006). *Country Synthesis Report on Urban Air Quality Management*. Malaysia: Asian Development Bank (ADB) and the Clean Air Initiative for Asian Cities (CAI-Asia) Centre.
- Awang, M., Jaafar, A. B., Abdullah, A. M., Ismail, M., Hassan, M. N., Abdullah, R., & Noor, H. (2000). Air quality in Malaysia: impacts, management issues and future challenges. *Respirology (Carlton, Vic.)*, 5(2000), 183-96.
- Basu, A. (1971). Fitting of a Markov chain model for daily rainfall data at Calcutta. *Indian Journal of Meteorology and Geophysics*, 22, 67-74.
- Berchtold, A., & Raftery, A. (2002). The Mixture Transition Distribution Model for High-Order Markov Chains and Non-Gaussian Time Series. *Statistical Science*, *17*(3), 328-356.
- Chaloulakou, A., Grivas, G., & Spyrellis, N. (2003). Neural network and multiple regression models for PM10 prediction in Athens: a comparative assessment. *Journal of the Air and Waste Management Association* (1995), 53, 1183-1190. http://doi.org/10.1080/10473289.2003.10466276
- Chin, E. H. (1977). Modeling daily precipitation occurrence process with Markov Chain. *Water Resources Research*, *13*(6), 949-956.
- Dahale, S. D., Panchawagh, N., Singh, S. V., Ranatunge, E. R., & Brikshavana, M. (1994). Persistence in rainfall occurrence over Tropical south-east Asia and equatorial Pacific. *Theoretical and Applied Climatology*, 49(1), 27-39. http://doi.org/10.1007/BF00866286
- Deni, S. M., Jemain, A. A., & Ibrahim, K. (2009). Fitting optimum order of Markov chain models for daily rainfall occurrences in Peninsular Malaysia. *Theoretical and Applied Climatology*, 97, 109-121.
- DoE. (2013). *Malaysia Environmental Quality Report 2013*. Kuala Lumpur: Department of Environment. Ministry of Natural Resources and Environment, Malaysia.
- DoS. (2013). Compendium of Environment Statistics. Malaysia: Department of Statistics, Malaysia.
- EPA. (2001). Latest Findings on National Air Quality: 2000 Status and Trends. United State Environmental Protection Agency.
- Gabriel, K. R., & Neumann, J. (1962). A Markov Chain Model for Daily Rainfall Occurrence at Tel Aviv. *Quarterly Journal of the Royal Meteorological Society*, 88(375), 90-95.
- Haan, C. T., Allen, D. M., & Street, J. O. (1976). A Markov Chain Model of daily rainfall. *Water Resources Research*, *12*(3), 443-449. http://doi.org/10.1029/WR012i003p00443
- Jimoh, O. D., & Webster, P. (1996). The optimum order of a Markov chain model for daily rainfall in Nigeria. *Journal of Hydrology*, 185, 45–69.
- Junninen, H., Niska, H., Tuppurainen, K., Ruuskanen, J., & Kolehmainen, M. (2004). Methods for Imputation of Missing Values in Air Quality Data Sets. Atmospheric Environment, 38(18), 2895-2907.

- Latif, M. T., Dominick, D., Ahamad, F., Khan, M. F., Juneng, L., Hamzah, F. M., & Nadzir, M. S. M. (2014). Long Term Assessment of Air Quality from A Background Station on the Malaysian Peninsula. Science of the Total Environment, 482-483(2), 336-348.
- Little, J., & Rubin, D. (2002). Statistical Analysis with Missing Data. New York: Wiley.
- Mangaraj, A. K., Sahoo, L. N., & Sukla, M. K. (2013). A Markov Chain Analysis of Daily Rainfall. *Journal of Reliability and Statistical Studies*, 6(1), 77-86.
- Moon, S. E., Ryoo, S. B., & Kwon, J. G. (1994). A Markov chain model for daily precipitation occurrence in South Korea. *International Journal of Climatology*, 14, 1009-1016.
- Paschalidou, A. K., Karakitsios, S., Kleanthous, S., & Kassomenos, P. A. (2011). Forecasting hourly PM10 concentration in Cyprus through artificial neural networks and multiple regression models: Implications to local environmental management. *Environmental Science and Pollution Research*, 18(2), 316-327.
- Pires, J. C. M., Martins, F. G., Sousa, S. I. V., Alvim-Ferraz, M. C. M., & Pereira, M. C. (2008). Prediction of the Daily Mean PM₁₀ Concentrations Using Linear Models. *American Journal of Environmental Sciences*, 4(5), 445-453.
- Rahimi, J., Bazfarshan, J., & Rahimi, A. (2011). Study of Persistence of Days Infected Pollutant Particulate Matter (PM₁₀) in City of Tehran Using Markov Chain Model. In 13th Conference on Atmospheric Chemistry (p. 231).
- Skuriat-Olechnowska, M. (2005). Statistical inference and hypothesis testing for Markov chains with Interval Censoring. Delft University of Technology. Retrieved from http://www.ewi.tudelft.nl/fileadmin/Faculteit/EWI/Over_de_faculteit/Afdelingen/Applied_Mathematics/Risico_en_Beslissings_Analyse/Theses/MSkuriat thesis.pdf
- Ul-Saufie, A. Z., Yahaya, A. S., Ramli, N. A., & Hamid, H. A. (2012). Robust Regression Models for Predicting PM₁₀ Concentration in an Industrial Area. *International Journal of Engineering and Technology*, 2(3), 364-370.
- Yusof, N. F. F. M., Ramli, N. A. R., & Yahaya, A. S. (2011). Extreme Value Distribution for Prediction of Future PM₁₀ Exceedences. *International Journal of Environmental Protection*, 1(4), 28-36.