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Design of the Side Sensitive Group Runs Chart with Estimated Parameters Based on Expected Average Run Length

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ABSTRACT

The assumption when constructing a control chart is that the process parameters, i.e. mean and standard deviation, are known. Nevertheless, this assumption is not realistic in practical situations. In most of the application of a control chart, the mean and standard deviation are unknown and are estimated from an in-control Phase-I samples. When the process parameters are estimated, the control chart performs differently compared with the corresponding chart with known process parameters because of the variability of estimators. The usual practice to evaluate the performance of a control chart is to use the average run length (ARL). The ARL is the average number of samples plotted on a control chart before an out-of-control signal is detected. In addition, the expected ARL (EARL) is used as a performance measure for the random process mean shift. In this article, the performance of the side sensitive group runs (SSGR) chart with known and estimated process parameters are studied and examined in terms of ARL and EARL.

Keywords: Estimated parameters, expected average run length, side sensitive group runs

INTRODUCTION

The control chart was introduced by Walter A. Shewhart in 1924 to monitor and determine whether a process is in statistical control (Montgomery, 2012) and if it is, the process is seen as conforming. This indirectly ensures the quality of output. Control charts are widely used in various fields, for example, manufacturing and the service industry. Therefore, the control chart has been recognised as one of the seven magnificent tools in Statistical Process Control (SPC).

The Shewhart chart has been widely used to monitor the process. However, the major shortcoming of the Shewhart chart is lack of sensitivity towards small and moderate process mean shifts (Sanusi, Abujiya, Riaz, & Abbas, 2017). In view of this, the Shewhart chart has been studied extensively to enhance

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the sensitivity of the control chart towards small and moderate process mean shift. For example, synthetic chart (Wu & Spedding, 2000a), group run (GR) chart (Gadre & Rattihalli, 2004) and side sensitive group run (SSGR) chart (Gadre & Rattihalli, 2007).

An indispensable assumption when designing a control chart is that process parameters, such as the mean and the standard deviation, are assumed known (Chen, Birch, & Woodall, 2016). However, in many applications of the control chart, the process parameters are unknown. Thus, the process parameters are usually estimated from an in-control Phase-I samples. Woodall and Montgomery (1999, 2014) pointed out that when the process parameters are estimated, the performance of the control chart will differ from the known process parameters case due to estimation error. Therefore, Psarakis, Vyniou and Castagliola (2014) emphasised that it is essential to study the performance of a control chart when process parameters are unknown. Jensen, Jones-Farmer, Champ and Woodall (2006), Saleh, Mahmoud, Jones-Farmer, Zwetsloot & Woodall (2015), You, Khoo, Castagliola and Ou, (2015) and Shepherd, Champ and Rigdon (2016), to name a few, examined the performance of control charts with unknown process parameters.

The performance of a control chart is an important characteristic to consider because it influences the decision on the use of the control chart. A well-known and common performance measure is average run length (ARL) (Chakraborti, 2007). The ARL indicates, on the average, how many samples need to be plotted before an out-of-control signal is detected. The computation of the ARL is based on the particular mean shift provided.

Nevertheless, there are situations when the practitioner is unable to identify the shift of a process (Teoh, Chong, Khoo, Castagliola, & Yeong, 2017). Moreover, the practitioners may not have historical knowledge or experience on the process to determine the shift of a process (Castagliola, Celano, & Psarakis, 2011). Determining the particular mean shift will result in decision inaccuracy of the process if a different mean shift occurs in the process. In view of this, expected average run length (EARL) is introduced in this paper to evaluate and design the SSGR chart. The EARL is obtained by integrating over the density function of the shift size.

In this paper, the SSGR chart with estimated process parameters will be investigated when the process mean shift is random. Moreover, the performance of the control chart will be compared with the particular process mean shift for the known and estimated process parameters case. This approach is to convince the practitioners to implement EARL when the shift size may not be known in advance.

METHODS

The SSGR chart developed by Gadre and Rattihalli (2007) comprises a Shewhart sub-chart and an extended version of the conforming run length (CRL) sub-chart. The Shewhart sub-chart is designed with two control limits, the LCL and UCL. Meanwhile, the CRL sub-chart has a single control limit, L, which is a specified positive integer.

The operation of the SSGR chart is explained as follows:

(1) When a sample falls below the LCL or above the UCL, the SSGR chart indicates this is a nonconforming sample.

(2) Further investigation using the CRL sub-chart is necessary to determine the state of the process.

(3) The r^{th} CRL value will be computed, i.e. CRL_r, for $r = 1, 2, \ldots$ is denoted as the number of conforming samples plotted on the Shewhart sub-chart between the r^{th} and $(r\text{-}1)^{th}$ nonconforming samples.

(4) The SSGR chart declares an out-of-control if
   (i) $CRL_1 \leq L$ or
   (ii) $CRL_r \leq L$ and $CRL_{r+1} \leq L$, for $r = 2, 3, \ldots$ and that both CRL_r and CRL_{r+1} fall on the same side of the Shewhart sub-chart.

It should be emphasised that when a sample falls outside the control limits of the Shewhart sub-chart, the SSGR chart does not signal an out-of-control status immediately. Instead, it just indicates a nonconforming sample. Further investigation using the CRL sub-chart is required before an out-of-control status is signalled.

When the process parameters are assumed known, the LCL and UCL are

$$LCL = \mu_0 - \frac{H}{\sqrt{n}}\sigma_0 \quad [1]$$

and

$$UCL = \mu_0 + \frac{H}{\sqrt{n}}\sigma_0, \quad [2]$$

where H is the design constant. The probability of a conforming sample on the Shewhart sub-chart is $A = \Pr\left(\bar{X} \in [LCL, UCL]\right)$, i.e.

$$A = \Pr\left(\mu_0 - \frac{H}{\sqrt{n}}\sigma_0 \leq \bar{X} \leq \mu_0 + \frac{H}{\sqrt{n}}\sigma_0\right). \quad [3]$$

Following simplification, A reduces to

$$A = \Phi\left(H - \delta\sqrt{n}\right) - \Phi\left(-H - \delta\sqrt{n}\right). \quad [4]$$

Here, $\Phi(.)$ is the standard normal cumulative distribution function (cdf).

Let B denote the probability that a sample on the Shewhart sub-chart is nonconforming, i.e.

$$B = \Pr\left(\bar{X} \notin [LCL, UCL]\right) = 1 - A. \quad [5]$$

In addition, the probability of an event $CRL_r \leq L$ is

$$C = 1 - (1 - B)^L. \quad [6]$$

Also, the conditional probability $k = k = \Pr\left(\bar{X} > UCL \middle| \bar{X} \notin [LCL, UCL]\right)$ for taking into account the side sensitivity aspect is

$$k = \frac{\Pr\left(\bar{X} > UCL\right)}{B} = \frac{1 - \Phi\left(H - \delta\sqrt{n}\right)}{B}. \qquad [7]$$

Finally, the ARL formula for the SSGR chart is (Gadre & Rattihalli, 2007)

$$\text{ARL} = \frac{1 - k(1-k)C^2}{BC^2[1 + k(1-k)(C-2)]}. \qquad [8]$$

The evaluation using ARL requires the shift size to be determined in advance. Unfortunately, this is not a practical situation. Therefore, the EARL can be employed in place of the ARL. The EARL is computed as

$$\text{EARL} = \int_{\delta_{\min}}^{\delta_{\max}} f_\delta\left(\delta\right) \text{ARL} d\delta. \qquad [9]$$

It is the expected value of the ARL integrated over the density function,$f_\delta(\delta)$ of the mean shift size in the process, i.e. δ. In this paper, two interval combinations of random shift sizes are set:$(\delta_{\min}, \delta_{\max}) = (0.1,1.0)$ and $(\delta_{\min}, \delta_{\max}) = (1.0,2.0)$, to evaluate the performance of the SSGR chart.

In a real situation, the process parameters are rarely known. Hence, the process parameters are usually estimated from m in-control Phase-I samples with each of size n. An estimator of μ_0 is

$$\hat{\mu}_0 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} Y_{i,j} \qquad [10]$$

and an estimator of σ_0 is

$$\hat{\sigma}_0 = \sqrt{\frac{1}{m(n-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(Y_{i,j} - \bar{Y}_i\right)^2}, \qquad [11]$$

where the sample mean is $\bar{Y}_i = \frac{1}{n}\sum_{j=1}^{n} Y_{i,j}$.

Based on the estimated process parameters, i.e. $\hat{\mu}_0$ and $\hat{\sigma}_0$ from the Phase-I parameter estimation, the LCL and UCL are

$$LCL = \hat{\mu}_0 - \frac{H'}{\sqrt{n}}\hat{\sigma}_0 \qquad [12]$$

and

$$UCL = \hat{\mu}_0 + \frac{H'}{\sqrt{n}}\hat{\sigma}_0, \qquad [13]$$

respectively, with H' being the design constant for the Shewhart sub-chart with estimated process parameters. Let $\hat{A}$ represent the probability that a sample is conforming on the Shewhart sub-chart and is computed as

$$\hat{A} = \Pr\left(\hat{\mu}_0 - \frac{H'}{\sqrt{n}} \hat{\sigma}_0 \le \bar{X} \le \hat{\mu}_0 + \frac{H'}{\sqrt{n}} \hat{\sigma}_0 \right). \quad [14]$$

By defining $V = \left(\hat{\mu}_0 - \mu_0\right)\frac{\sqrt{n}}{\sigma_0}$ and $W = \frac{\hat{\sigma}_0\sqrt{n}}{\sigma_0}$, the Equation [14] simplifies to (You, Khoo, Castagliola, & Ou, 2015)

$$\hat{A} = \Phi\left(V + \frac{H'}{\sqrt{n}} W - \delta\sqrt{n} \right) - \Phi\left(V - \frac{H'}{\sqrt{n}} W - \delta\sqrt{n} \right). \quad [15]$$

Because $\hat{\mu}_0 \sim N\left(\mu_0, \frac{\sigma_0^2}{mn}\right)$, it can be deduced that $V \sim N\left(0, \frac{1}{m}\right)$. Then, the probability density function (pdf) of V is

$$f_V\left(v \middle| m\right) = f_N\left(v \middle| 0, \frac{1}{m}\right), \quad [16]$$

where f_N is the pdf of the normal distribution with mean 0 and variance $\frac{1}{m}$. According to Zhang, Castagliola, Wu and Khoo (2011), it can be shown that $W^2 = \frac{\hat{\sigma}_0^2 n}{\sigma_0^2} \sim \gamma\left(\frac{m(n-1)}{2}, \frac{2n}{m(n-1)}\right)$. Hence, the pdf of W is

$$f_W\left(w \middle| m, n\right) = 2w f_\gamma\left(w^2 \middle| \frac{m(n-1)}{2}, \frac{2n}{m(n-1)}\right), \quad [17]$$

where f_γ is the pdf of the gamma distribution with parameters $\frac{m(n-1)}{2}$ and $\frac{2n}{m(n-1)}$.

The probability that a sample is nonconforming on the Shewhart sub-chart is

$$\hat{B} = 1 - \hat{A}. \quad [18]$$

In a similar manner, the probability of an event $\text{CRL}_r \le L'$ and the conditional probability, $\Pr\left(\bar{X} > UCL \middle| \bar{X} \notin \left[LCL, UCL\right]\right)$ are

$$\hat{C} = \Pr\left(\text{CRL}_r \le L'\right) = 1 - \left(1 - \hat{B}\right)^{L'} \quad [19]$$

and

$$\hat{k} = \frac{1-\Phi\left(V+\frac{H'}{\sqrt{n}}W-\delta\sqrt{n}\right)}{\hat{B}}, \qquad [20]$$

respectively, with L' being the control limit of the CRL sub-chart with estimated process parameters.

Finally, the ARL of the SSGR chart with estimated process parameters is

$$\text{ARL} = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \frac{1-\hat{k}\left(1-\hat{k}\right)\hat{C}^2}{\hat{B}\hat{C}^2\left(1+\hat{k}\left(1-\hat{k}\right)\left(\hat{C}-2\right)\right)} f_V\left(v|m\right) f_w\left(w|m,n\right) dwdv. \qquad [21]$$

Furthermore, the EARL of the SSGR chart with estimated process parameters is

$$\text{EARL}=\int_{\delta_{\min}}^{\delta_{\max}}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} f_\delta\left(\delta\right)\text{ARL} f_V\left(v|m\right) f_W\left(w|m,n\right) dwdvd\delta. \qquad [22]$$

where ARL can be obtained from Equation (8) by replacing B, C and k with $\hat{B}$, $\hat{C}$and $\hat{k}$, respectively.

RESULTS AND DISCUSSION

Table 1 presents the ARL performance for the SSGR chart. Let ARL_0 and ARL_1 represent the in-control ARL and out-of-control ARL respectively. For comparison purposes, we include computations of ARL_1s for the known process parameters chart and denote it as $m = +\infty$. Meanwhile, $m = \{30, 50, 80, 200, 500\}$ represents the cases when process parameters are estimated.

Different combinations of number of in-control Phase-I samples, $m \in \{30, 50, 80, 200, 500, +\infty\}$, sample sizes, $n \in \{3, 5, 7, 9\}$ and process mean shift sizes, $\delta = \{0.1, 0.3, 0.7, 1.1, 1.5, 2.0\}$are considered in Table 1. The results of in columns 4 – 9 are evaluated using the optimal pair (*H, L*) of the chart with known process parameters, i.e. the optimal pair (*H, L*) displayed in column 3. The optimal pair (*H, L*) will give an intended $\text{ARL}_0 = 370.4$ when $\delta = 0$ for $m = +\infty$.

For example, when n = 5, $\delta = 0.3$, the optimal pair for minimising ARL_1 is (*H, L*) = (2.2515, 22), for the SSGR chart with known process parameters ($m = +\infty$). The corresponding smallest $\text{ARL}_1 = 32.13$ is obtained while achieving the desired $\text{ARL}_0 = 370.4$ when $m = +\infty$. This optimal pair yields $\text{ARL}_1 \in \{60.59, 45.72, 39.65, 34.82, 33.16\}$ for $m \in \{30, 50, 80, 200, 500\}$. Note that when the process parameters are estimated from a small Phase-I samples, i.e. $m = 30$ with each of size $n = 5$, the corresponding ARL_1 (=60.59) differs from the ARL_1 (=32.13) when process parameters are assumed to be known. This shows that the performance of the SSGR chart deteriorates significantly when the process parameters are estimated due to the effect of parameter estimations. Nevertheless, when $m = 200$, the corresponding ARL_1 (=34.82) is almost similar to the ARL_1 when process parameters are known. These results reveal that more than 80 samples are required for the SSGR chart with estimated process parameters to perform satisfactorily with the same chart with known process parameters.

Table 1
ARL_1s for n = {3, 5, 7, 9} with different combinations of (m, δ), based on the optimal pair (H, L) corresponding to the known process parameters case when ARL_0 *= 370.4*

n	*δ*	*(K, L)*	*m*					
			30	50	80	200	500	+ ∞
3	0.1	(2.4913, 63)	1425.24	635.25	439.44	319.14	283.85	263.24
	0.3	(2.3250, 30)	228.44	119.80	89.01	68.97	62.95	59.41
	0.7	(1.9948, 8)	8.72	7.46	6.92	6.48	6.32	6.22
	1.1	(1.8025, 4)	2.20	2.12	2.08	2.05	2.03	2.02
	1.5	(1.5953, 2)	1.28	1.27	1.26	1.25	1.25	1.25
	2.0	(1.5953, 2)	1.04	1.04	1.04	1.03	1.03	1.03
5	0.1	(2.4696, 57)	439.47	327.96	281.04	241.28	226.77	217.43
	0.3	(2.2515, 22)	60.59	45.72	39.65	34.82	33.16	32.13
	0.7	(1.8660, 5)	3.57	3.39	3.30	3.21	3.18	3.16
	1.1	(1.7185, 3)	1.36	1.35	1.34	1.33	1.33	1.33
	1.5	(1.5953, 2)	1.05	1.05	1.05	1.04	1.04	1.04
	2.0	(1.5953, 2)	1.00	1.00	1.00	1.00	1.00	1.00
7	0.1	(2.4537, 53)	299.81	248.96	223.56	199.29	189.72	183.38
	0.3	(2.1886, 17)	31.74	26.22	23.78	21.72	20.99	20.52
	0.7	(1.8025, 4)	2.29	2.23	2.20	2.17	2.15	2.15
	1.1	(1.5953, 2)	1.14	1.13	1.13	1.13	1.12	1.12
	1.5	(1.5953, 2)	1.01	1.01	1.01	1.01	1.01	1.01
	2.0	(1.5953, 2)	1.00	1.00	1.00	1.00	1.00	1.00
9	0.1	(2.4364, 49)	238.61	205.82	187.80	169.55	162.16	157.23
	0.3	(2.1401, 14)	20.21	17.47	16.21	15.11	14.72	14.46
	0.7	(1.7185, 3)	1.76	1.73	1.71	1.69	1.69	1.68
	1.1	(1.5953, 2)	1.06	1.05	1.05	1.05	1.05	1.05
	1.5	(1.5953, 2)	1.00	1.00	1.00	1.00	1.00	1.00
	2.0	(1.5953, 2)	1.00	1.00	1.00	1.00	1.00	1.00

In a real application, practitioners may not know the shift size in advance. Hence, if a practitioner considers a particular shift size, δ and employs the corresponding optimal pair, the performance of the SSGR chart will be significantly affected if a different shift size actually occurs in the process. In view of this, EARL is crucial to adopt in place of ARL for designing and evaluating the SSGR chart.

The $EARL_0$ and $EARL_1$ denote the in-control and out-of-control EARL respectively. Table 2 displays the $EARL_1$ computed using the same (*m, n*) combinations considered in Table 1. The shift interval (δ_{min}, δ_{max}) = (0.1, 1.0) and (δ_{min}, δ_{max}) = (1.0, 2.0) are considered here, so that it includes the exact shifts in the Table 1. For instance, the shift interval (δ_{min}, δ_{max}) = (0.1, 1.0) includes the shifts δ = {0.1, 0.3, 0.7}. Similarly, $(\delta_{min}, \delta_{max})$ = (1.0, 2.0) in Table 2 is considered to include δ = {1.1,1.5,2.0).

Table 2

$EARL_1$s for n = {3, 5, 7, 9} with Different Combinations of (m, δ_{min}, δ_{max}), based on the optimal pair (H, L) corresponding to the known process parameters case when $EARL_0$ = 370.4

n	δ_{min}	δ_{max}	*(K, L)*	*m*					
				30	50	80	200	500	$+\infty$
3	0.1	1.0	(2.2821, 25)	143.27	81.67	62.56	49.33	45.17	42.69
	1.0	2.0	(1.7185, 3)	1.47	1.44	1.43	1.42	1.41	1.41
5	0.1	1.0	(2.2515, 22)	50.03	39.14	34.28	30.09	28.58	27.62
	1.0	2.0	(1.5953, 2)	1.12	1.12	1.11	1.11	1.11	1.11
7	0.1	1.0	(2.2403, 21)	31.56	26.43	23.89	21.57	20.70	20.13
	1.0	2.0	(1.5953, 2)	1.04	1.04	1.04	1.04	1.04	1.04
9	0.1	1.0	(2.2159, 19)	23.01	19.83	18.18	16.63	16.03	15.65
	1.0	2.0	(1.5953, 2)	1.02	1.01	1.01	1.01	1.01	1.01

For illustration, when n = 3, δ_{min} = 0.1 and δ_{max} =1.0 , the optimal pair (H, L) = (2.2821, 25) yields the smallest $EARL_1$ = 42.69 when the process parameters are known. Meanwhile, this optimal pair (H, L) produces the intended $EARL_0$ = 370.4. However, the $EARL_1$ are 143.27, 81.67, 62.56, 49.33 and 45.17 for m = 30, 50, 80, 200 and 500, respectively. Comparing these with $EARL_1$ when process parameters are known (i.e. $EARL_1$ = 42.96), the $EARL_1$ values for the SSGR chart with estimated process parameters are significantly larger than the corresponding value of the same chart with known process parameters, especially when m is small. This provides clear insight that the $EARL_1$ value of the SSGR chart with estimated process parameters converge to the known process parameters case when m increases. This phenomenon of the SSGR chart is similar to that presented in Table 1 when ARL_1 is employed as a performance measure.

Furthermore, it is observed that the ARL_1 values computed using the optimal pair (H, L) for minimising $EARL_1$ in Table 2 are quite similar to the ARL_1 values computed using the optimal pair (H, L) for minimising ARL_1 in Table 1, as long as $\delta \in (\delta_{min}, \delta_{max})$.

For instance, when n = 5, δ_{min} = 0.1 and δ_{max} = 1.0 the optimal pair (H, L) = (2.2515, 22) and the corresponding $EARL_1$ = 27.62 when $m = +\infty$. By considering δ = 0.7 (i.e. $\delta \in (\delta_{min}, \delta_{max})$), the ARL_1 values are computed as 4.43, 4.27, 4.19, 4.12, 4.09 and 4.08 for m = 30, 50, 80, 200, 500 and $+\infty$ using (H, L) = (2.2515, 22). These ARL_1 values are quite similar to those in Table 1 when n = 5 and δ = 0.7, although the optimal pair (H, L) are different. In Table 1, when n = 5 and δ = 0.7, the optimal pair is (H, L) = (1.8660, 5). This suggests that the optimal pair (H, L) obtained by minimising $EARL_1$ is reliable to compute the ARL_1, as long as $\delta \in (\delta_{min}, \delta_{max})$. Note that the application of $EARL_1$ is more reasonable because practitioners may not have knowledge or experience to determine the exact process shift. Moreover, practitioners may not know the particular process shift that will occur in the process.

CONCLUSION

In summary, measures like ARL require practitioners to determine the exact shift size. In practical application, the particular shift size is usually unknown in advance. Hence, in this paper, the SSGR chart is designed to minimise $EARL_1$ when process parameters are known. The results reveal the optimal pair (*H, L*) obtained by minimising $EARL_1$ when process parameters are known can be used to compute ARL_1, as long as $\delta \in (\delta_{min}, \delta_{max})$.

Moreover, the study results indicate that a large number of Phase-I samples are required for the SSGR chart with estimated process parameters to perform favourably compared with the same chart with known process parameters case. This study can be extended to propose the optimal pair by minimising $EARL_1$ for the SSGR chart when process parameters are estimated. Furthermore, the expected value of the summary measure can be examined, for example, the expected value for percentile, when the exact shift size is unknown in advance.

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