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A Goal Programming Model for Portfolio Optimisation Problem in Fuzzy Environment

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ABSTRACT

Portfolio optimisation is one of the most crucial issues in investment decision-making and has received considerable attention from researchers and practitioners. Traditionally, the portfolio optimisation models are formulated based on the assumption that investors have complete information on the distribution of random returns. However, in real life case, this is not possible since decisions have to be made under uncertainty. This paper deals with a fuzzy portfolio optimisation problem in which returns and turnover rates of securities are represented by fuzzy variables. A goal programming model is proposed to optimise three objectives: maximisation of portfolio return, maximisation of liquidity and minimisation of the portfolio risk. The cardinality constraints, floor and ceiling constraints are also taken into consideration. Finally, a numerical experiment using real data is conducted to demonstrate the applicability of the model.

Keywords: Portfolio optimisation, goal programming, fuzzy portfolio, multi-objective

INTRODUCTION

Over the past few decades, portfolio optimisation problem has become one of the most interesting topics in the field of financial management and investment. Its basic formulation is based on selecting a set of securities among a number of available ones which can best meet the investor's goal. The first portfolio selection model was introduced by Markowitz in 1952. The model, also known as mean-variance model, has become a basis for the development of modern portfolio theory. In Markowitz's approach, the problem is formulated based upon two criteria: the portfolio return and the portfolio risk. The portfolio return is described by the mean return of the securities while the portfolio risk is quantified by the variance

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of returns between the securities. The mean-variance model assumes that a rational investor wishes to maximise the portfolio expected return for a given level of risk, or alternatively, the investor wants the lowest portfolio risk for a given level of expected return.

Although the mean-variance model has been widely accepted for its theoretical reputations, it has not been applied extensively to construct a large-scale portfolio (Konno & Yamazaki, 1991). This is mainly due to the difficulty of handling a large-scale quadratic programming problem with a dense variance-covariance matrix. In order to alleviate this difficulty, several attempts have been made to transform the quadratic problem into a linear one. Konno and Yamazaki (1991) proposed mean absolute deviation as a new measure of risk to replace the variance in Markowitz's model. The authors showed that the mean absolute deviation model is equivalent to the mean-variance model when returns are assumed to have a normal distribution. Motivated by Konno and Yamazaki's work, Speranza (1993) proposed mean semi-absolute deviation to evaluate risk in portfolio selection model. This risk measure describes the preferences of investors in a more realistic way since it only considers return below the mean. From a computational point of view, Speranza (1993) showed that the semi-absolute deviation reduced the number of required constraints by half in comparison with the mean absolute deviation.

In the above-cited works, portfolio return and risk are considered as the only main factors that impact an investor's decision. However, many have argued that some of the relevant information for selecting a portfolio can never be completely captured in terms of these two criteria. There are other considerations that might be important to investors. As a result, numerous portfolio optimisation models that consider criteria other than risk and return have been developed in recent years. Ehrgott et al. (2004), for example, extended the mean-variance model by formulating a hierarchy of objectives, which breaks down risk and return into five sub-objectives and employed a multi-criteria decision-making method to solve the problem. Gupta et al. (2008) proposed a portfolio optimisation model based on semi-absolute deviation function. Their model considers multiple objectives which are short term return, long term return, annual dividend, risk, and liquidity. Li and Xu (2013) presented a multi-objective portfolio selection model with fuzzy random returns for investors. Their model optimizes three objectives, namely, return, risk and liquidity.

One of the most popular and promising techniques to handle portfolio optimisation problem with multiple and conflicting objectives is goal programming. Developed by Charnes and Cooper in 1961, it focuses on minimising deviations between the realised goal and the desired target. The minimisation process can be achieved using different approaches, each one leads to several variants of goal programming. Unlike linear programming that seeks for an optimal solution, goal programming attempts to look for a satisfactory solution that comes as close as possible to the desired goals. According to Arenas-Parra et al. (2010), the main advantage of goal programming approach is it provides decision makers with enough flexibility to include numerous variations of constraints and goals into a model.

Goal programming approach which is a branch of multi-objective optimisation has been extensively and successfully applied to formulate portfolio selection problems(see Pendaraki et al., 2005; Sharma & Sharma, 2006; Stoyan & Kwon, 2011; Tamiz et al., 2013). Most of the models however, assume that the future returns of security are dependent on random variables.

In practical investment, many other uncertain factors affect the stock markets such as economy, policies, laws and regulations (Liu & Zhang, 2013). Under such situations, fuzzy set theory initiated by Zadeh (1965) has become a useful tool in managing the vagueness and ambiguity of security returns. In recent years, much research has been done on portfolio optimisation problems in fuzzy environment. For instance, Carlsson et al. (2002) introduced a possibilistic approach for selecting portfolios with highest utility score based on the assumptions that the security returns are characterised by trapezoidal fuzzy numbers. Vercher et al. (2007) presented portfolio optimisation models under downside risk approach using interval-valued probabilistic and possibilistic means. Zhang et al. (2007) proposed two portfolio selection models based on lower and upper possibilistic means and possibilistic variances of fuzzy numbers. In addition, Huang (2011) proposed two credibility-based minimax mean-variance models for fuzzy portfolio selection problem in the situation that each security return belongs to a certain class of fuzzy variables.

This paper deals with a portfolio optimisation problem in fuzzy environment. We propose a goal programming model by considering three objectives which are minimisation of portfolio risk, maximisation of portfolio return and maximisation of liquidity. The returns and turnover rates of securities are characterised by fuzzy variables. In addition, the model includes practical constraints such as cardinality constraints, floor and ceiling constraints.

The remaining part of this paper is organised as follows: Section 2 introduces some basic concepts of fuzzy numbers while Section 3 presents the formulation of a fuzzy goal programming model for portfolio optimisation problem with practical constraints. Section 4 illustrates a numerical example of the proposed model along with the corresponding results. The final section (Section 5), ends with some concluding remarks.

PRELIMINARIES

In this section, we define some basic concepts about fuzzy numbers and the expected value of a fuzzy number which will be used in the following sections. A fuzzy number *A* is a fuzzy set of the real line $\Re$, characterised by means of a membership function $\mu_A(x)$ which is upper semi-continuous and satisfies the condition $sup_{x\in\Re}\ \mu_A(x)=1$, and whose γ-cuts, for $0 \le \gamma \le 1$: $[A]^\gamma = \{x \in \Re : \mu_A(x) \ge \gamma\}$, are convex sets.

A fuzzy number *A* are called trapezoidal with tolerance interval [*a*; *b*], left width α and right width β if its membership function has the following form:

$$\mu_A(x)=\begin{cases} 1-\dfrac{a-x}{\alpha} & \text{if} \quad a-\alpha \le x \le a \\ 1 & \text{if} \quad a \le x \le b \\ 1-\dfrac{x-b}{\beta} & \text{if} \quad a \le x \le b+\beta \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and it can be written as $A = (a, b, \alpha, \beta)$. Then, the γ-level sets of A can easily be calculated as

$$[A]^{\gamma} = [a - (1-\gamma)\alpha, b + (1-\gamma)\beta], \qquad \forall_{\gamma} \in [0,1],$$

Let A be a fuzzy number with $[A]^{\gamma} = [a_1(\gamma), a_2(\gamma)]$, $\gamma \in [0,1]$. Carlsson and Fullér (2001) defined the lower and upper possiblistic expected value of fuzzy number A as

$$\widetilde{E}_*(A) = 2\int_0^1 \gamma a_1(\gamma) d\gamma \tag{2}$$

$$\widetilde{E}^*(A) = 2\int_0^1 \gamma a_2(\gamma) d\gamma \tag{3}$$

The interval-valued and crisp possibilistic expected values of fuzzy number A are defined as (Carlsson & Fullér, 2001)

$$\widetilde{E}(A) = [\widetilde{E}_*(A), \widetilde{E}^*(A)] \tag{4}$$

$$E(A) = \frac{\widetilde{E}_*(A) + \widetilde{E}^*(A)}{2} \tag{5}$$

MODEL FORMULATION

Assume that there are N securities available for trading in stock market. The return rates of the N securities are denoted as trapezoidal fuzzy numbers. The reason is that it can represent quite well the empirical distribution of security returns which have asymmetric and fat tails.

Following Vercher et al. (2007), the tolerance interval [a, b], left width α and right width β of the fuzzy return for every security j are computed based on the percentiles of data distribution of the security return. Using this approach, the imprecise estimation of the expected return of each security can be simply and clearly represented based on actual data. For ease of description, basic notations used in this study are defined in Table 1.

Table 1
Basic Notations

Index	
j	Set of securities, $j = 1, \ldots, N$
Decision variables	
x_j	Proportion invested in security j
q_j	Binary variable which will be 1 if any of security j is held and 0 otherwise and 0 otherwise
Parameters	
r_j	Return rate of security j
L_j	Turnover rate of security j
δ_j	Upper limits of the budget that can be invested in security j
υ_j	Lower limits of the budget that can be invested in security j
K	Desired number of securities to be included in the portfolio

Model's Objective

In the proposed multi-objective portfolio optimisation problem, the following objectives were considered.

Portfolio return. The portfolio return is the most practical objective which is usually used in portfolio optimisation models. In this paper, the return rates of the j^{th} asset are characterised by the trapezoidal fuzzy numbers $r_j = (a_j, b_j, \alpha_j, \beta_j)$ whose γ-level cuts are $[r_j]^\gamma = [a - (1-\gamma)\alpha, b + (1-\gamma)\beta]$ for $\forall_\gamma \in [0,1]$. Therefore, the total return of a portfolio $x = (x_1, x_2, \ldots, x_n)$ is the following trapezoidal fuzzy number

$$
\begin{aligned}
P &= \sum_{j=1}^{N} r_j x_j \\
&= \left(\sum_{j=1}^{N} a_j x_j, \sum_{j=1}^{N} b_j x_j, \sum_{j=1}^{N} \alpha_j x_j, \sum_{j=1}^{N} \beta_j x_j \right) \\
&= (P_l(x), P_u(x), C(x), D(x))
\end{aligned}
$$

The γ-level cuts of P are computed as

$$[P]^\gamma = [P_l(x) - (1-\gamma)C(x), P_u(x) + (1-\gamma)D(x)]$$

Based on the equation (2), (3), (4) and (5), we can obtain the lower and upper possibilistic means, the interval-valued and crisp possibilistic mean values of the portfolio return as follows:

$$\widetilde{E}_*(P) = 2\int_0^1 \gamma(P_l(x) - (1-\gamma)C(x))d\gamma$$

$$= P_l(x) - \frac{1}{3}C(x)$$

$$\widetilde{E}^*(A) = 2\int_0^1 \gamma(P_u(x) + (1-\gamma)D(x))d\gamma$$

$$= P_u(x) + \frac{1}{3}D(x)$$

$$\widetilde{E}(P) = \left[P_l(x) - \frac{1}{3}C(x), P_u(x) + \frac{1}{3}D(x)\right]$$

$$E(P) = \frac{1}{2}(P_l(x) + P_u(x)) + \frac{1}{6}(D(x) - C(x))$$

Thus, the possibilistic mean value of the return on portfolio $x = (x_1, x_2, \ldots, x_n)$ is given by

$$E(P) = \sum_{j=1}^{N}\left(\frac{1}{2}(a_j + b_j) + \frac{1}{6}(\beta_j - \alpha_j)\right)x_j \tag{6}$$

Portfolio risk. The portfolio risk is measured by the possibilistic semi-absolute deviation which is defined by Vercher et al. (2007) as

$$\widetilde{w}(P) = E(\max\{0, \widetilde{E}(P) - P\}) \tag{7}$$

Proposition 1: (Carlsson & Fullér, 2001) Let $r_j = (a_j, b_j, \alpha_j, \beta_j)$ be the trapezoidal return on the j^{th} asset, $j = 1, \ldots, N$, and let $P = (P_l(x), P_u(x), C(x), D(x))$ be the total return of the portfolio $x = (x_1, x_2, \ldots, x_n)$, then

a) $\max\{0, \widetilde{E}(P) - P\} = \left(0, P_u(x) - P_l(x) + \frac{D(x)}{3}, 0, C(x)\right)$

b) $\widetilde{W}(P) = E(\max\{0, \widetilde{E}(P) - P\}) = \left[0, P_u(x) - P_l(x) + \frac{C(x) + D(x)}{3}\right]$

Based on the Proposition 1, the interval-valued possibilistic semi-absolute deviation can be expressed as follows:

$$\widetilde{w}(P) = \left[0,\, P_u(x) - P_l(x) + \frac{C(x) + D(x)}{3}\right]$$

Thus, the crisp possibilistic semi-absolute deviation of the return associated with the portfolio $x = (x_1, x_2, \ldots, x_n)$ is given by

$$\begin{aligned} w(P) &= \frac{P_u(x) - P_l(x)}{2} + \frac{C(x) + D(x)}{6} \\ &= \sum_{j=1}^{N}\left(\frac{1}{2}\left(b_j - a_j\right) + \frac{1}{6}\left(\alpha_j + \beta_j\right)\right)x_j \end{aligned} \tag{8}$$

Liquidity. In practical investment, liquidity is also one of the main concerns for investors. Liquidity can be defined as the ability to easily sell a security without affecting its price in the market and without incurring a significant loss. It can be computed using turnover rate which is the proportion between the average stock traded at the market in the most recent month and the outstanding shares of that stock for that month. In this study, the turnover rates of security j are denoted by trapezoidal fuzzy numbers $l_j = (la_j, lb_j, l\alpha_j, l\beta_j)$ since they cannot be accurately predicted. The γ-level cuts of l_j are $[l_j]^\gamma = [la_j - (1-\gamma)l\alpha_j, lb_j + (1-\gamma)l\beta_j]$ for $\forall_\gamma \in [0,1]$. Thus, the liquidity of a portfolio $x = (x_1, x_2, \ldots, x_n)$ is the following trapezoidal fuzzy number

$$\begin{aligned} L &= \sum_{j=1}^{N} l_j x_j \\ &= \left(\sum_{j=1}^{N} la_j x_j, \sum_{j=1}^{N} lb_j x_j, \sum_{j=1}^{N} l\alpha_j x_j, \sum_{j=1}^{N} l\beta_j x_j\right) \\ &= (Q_l(x), Q_u(x), F(x), G(x)) \end{aligned}$$

Based on the equation (2), (3), (4) and (5), the interval-valued and crisp possibilistic mean values of the portfolio liquidity can be expressed as

$$\widetilde{E}(L) = \left[Q_l(x) - \frac{1}{3}F(x),\, Q_u(x) + \frac{1}{3}G(x)\right]$$

$$E(L) = \frac{1}{2}\left(Q_l(x) + Q_u(x)\right) + \frac{1}{6}\left(G(x) - F(x)\right)$$

Therefore, the crisp possibilistic mean value of the turnover rate of the portfolio can be expressed as follows:

$$E(L) = \sum_{j=1}^{N}\left(\frac{1}{2}\left(a_j + b_j\right) + \frac{1}{6}\left(l\beta_j - l\alpha_j\right)\right)x_j \quad (9)$$

Constraints

There are two types of constraints: theoretical and practical. The theoretical constraint is necessary in order to define the feasibility of a solution.

Budget constraint. Budget constraint is imposed in order to normalise the solution. It ensures that all the available capital are invested. This constraint can be written as follows:

$$\sum_{j=1}^{N} x_j = 1$$

Floor and ceiling constraints. Floor constraints are used in practice to avoid the cost of administrating very small portions of securities which will have a negligible influence on the portfolio's performance. Ceiling constraints are imposed to limit the excessive concentration of the portfolio to a specific security. By introducing a binary variable q_j, (equal to 1 if security j is in the portfolio and 0 otherwise) the constraint can be expressed as follows:

$$\upsilon_j q_j \le x_j \le \delta_j q_j \quad j = 1, 2, \ldots, N$$

Cardinality constraints. Cardinality constraints limit the total number of securities held in a portfolio. This constraint is imposed to simplify the management of the portfolio and to reduce transaction costs. This constraint is formulated as follows:

$$\sum_{j=1}^{N} q_j = K$$

Goal Programming

Basically, there are three variants in goal programming which are lexicographic, weighted and MinMax goal programming. Lexicographic goal programming approach assigns pre-emptive priority to different goals in order to minimise the sum of the unwanted deviation variables. The weighted goal programming approach assigns weights to goal deviations based on their relative importance and seeks to minimise the total weighted deviations of the goals. Finally, the MinMax goal programming attempts to minimise the largest unwanted deviation from the desired goals.

In this paper, we employed the lexicographic goal programming approach which requires ranking the goals in order of importance. A goal placed at the higher priority level is infinitely more important than a goal placed at the lower priority level. The problem is then solved by meeting the highest priority goal first as closely as possible before proceeding to the next priority goal. The procedure continues until the goal placed at the lowest priority level is solved. Using this approach, the solution achieved by a higher priority goal is never degraded by a lower priority goal.

The framework of pre-emptive goal programming model can be formulated as follows (Jones & Tamiz, 2010):

$$\text{Lex min } a = \left[h_1\left(d_i^-, d_i^+\right), \ldots, h_p\left(d_i^-, d_i^+\right)\right]$$

subject to,

$$f_i(x) + d_i^- - d_i^+ = g_i \qquad i = 1, \ldots, m$$

$$x \in F$$

$$x \geq 0$$

$$d_i^-, d_i^+ \geq 0 \qquad i = 1, \ldots, m$$

where d_i^- and d_i^+ are the negative and positive deviational variables attached to the goal *i*, ($i = 1, \ldots, m$), $f_i(x)$ is the mathematical expression of the i^{th} goal, g_i is the target value of goal *i*, *x* is the vector of the decision variables, *F* is a set of hard constraints that may exist in the model, h_s is the index set of goals placed in the s^{th} priority level and *a* is the lexicographic optimisation process.

Formulation of the Goal Programming Model

The first goal is to obtain the maximum possible return on the investment. This goal is formulated to minimise negative deviation. From equation (6), this goal can be expressed as follows:

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(a_j + b_j\right) + \frac{1}{6}\left(\beta_j - \alpha_j\right)\right)x_j + d_1^- - d_1^+ = g_1$$

where g_1 is the desired portfolio return. The second goal is to reduce the portfolio risk to a certain level. To achieve this goal, the positive deviation is minimised. Thus, from equation (8),

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(b_j - a_j\right) + \frac{1}{6}\left(\alpha_j + \beta_j\right)\right)x_j + d_2^- - d_2^+ = g_2$$

where g_2 is an acceptable level of risk. The third goal is to maximise the portfolio liquidity. This goal is formulated by minimising the negative deviation. From equation (9), this goal is given by

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(la_j + b_j\right) + \frac{1}{6}\left(l\beta_j - l\alpha_j\right)\right)x_j + d_3^- - d_3^+ = g_3$$

where g_3 is an acceptable level of liquidity. Based on the above discussion, the goal programming model for portfolio optimisation problem is formulated as follows:

Lex min $a = \left(d_1^-, d_2^+, d_3^-\right)$
subject to,

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(a_j + b_j\right) + \frac{1}{6}\left(\beta_j - \alpha_j\right)\right)x_j + d_1^- - d_1^+ = g_1$$

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(b_j - a_j\right) + \frac{1}{6}\left(\alpha_j + \beta_j\right)\right)x_j + d_2^- - d_2^+ = g_2$$

$$\sum_{j=1}^{N}\left(\frac{1}{2}\left(la_j + b_j\right) + \frac{1}{6}\left(l\beta_j - l\alpha_j\right)\right)x_j + d_3^- - d_3^+ = g_3$$

$$\sum_{j=1}^{N} x_j = 1$$

$$\upsilon_j q_j \le x_j \le \delta_j q_j \qquad j = 1, 2, \ldots, N$$

$$\sum_{j=1}^{N} q_j = K$$

$$x_j \ge 0 \qquad j = 1, 2, \ldots, N$$

$$d_i^-, d_i^+ \ge 0 \qquad j = 1, 2, \ldots, N$$

$$q_j \in \{0,1\} \qquad j = 1, 2, \ldots, N$$

NUMERICAL EXAMPLE

Data

In this study, the model was tested on 100 Shariah-compliant stocks listed on the main board of Bursa Malaysia. The distributions of the selected securities in the corresponding sectors are shown in Table 2.

Table 2
Number of stocks in the sample per sector

Sector	Number of Securities	Decision Variables
Consumer	14	$x_1 - x_{14}$
Industrial	33	$x_{15} - x_{47}$
Technology	4	$x_{48} - x_{51}$
Plantation	6	$x_{52} - x_{57}$
Construction	5	$x_{58} - x_{62}$
Properties	11	$x_{63} - x_{73}$
Trading and Services	23	$x_{74} - x_{96}$
Infrastructure	3	$x_{97} - x_{99}$
Finance	1	x_{100}
Total	100	

The historical data consist of closing prices and turnover rates of all securities starting from January 2008 until December 2012. All data are collected from Datastream software. Data of monthly closing prices were transformed to monthly rate of returns using the formula below:

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}}$$

where p_t and p_{t-1} are the stock prices at time t and $t-1$ respectively.

In this study, since the future return rates and turnover rates of the securities were assumed to be trapezoidal fuzzy numbers, the tolerance interval, left width and right width of the fuzzy numbers were approximated using sample percentiles method introduced by Vercher et al. (2007). First, we calculate the 5th, 40th, 60th and 95th percentiles of the samples using historical data. Then, we set the interval $[P_{40}, P_{60}]$ as the core and the quantities $P_{40} - P_5$ and $P_{95} - P_{60}$ as the left and right spreads respectively, where P_w is the w^{th} percentile of the sample. Thus, the possibility distribution of security j is obtained, that is, $a_j = P_{40}$, $b_j = P_{60}$, $\alpha_j = P_{40} - P_5$ and $\beta_j = P_{95} - P_{60}$.

In this example, the minimum investment in each stock is 3% and the maximum investment must be of 20%. We also assume that the acceptance level of b_1, b_2 and b_3 are 0.03, 0.05 and 0.10 respectively. In order to illustrate the effect of number of securities on the portfolio selection, we vary the value of K from 10 to 15. The problems were solved using MATLAB R2014a and the results are presented in Table 3.

Table 3
Return, Risk and Liquidity of Portfolio

	Objectives			Selected stocks
K	Return	Risk	Liquidity	
10	0.03	0.0495	0.10	$x_4 = 0.03$, $x_{10} = 0.03$, $x_{34} = 0.03$, $x_{40} = 0.0455$, $x_{52} = 0.03$, $x_{60} = 0.2$, $x_{61} = 0.0553$, $x_{69} = 0.2$, $x_{75} = 0.1792$, $x_{78} = 0.2$
11	0.03	0.05	0.10	$x_4 = 0.03$, $x_{10} = 0.03$, $x_{16} = 0.03$, $x_{32} = 0.0320$, $x_{52} = 0.03$, $x_{60} = 0.2$, $x_{61} = 0.03$, $x_{62} = 0.0784$, $x_{69} = 0.1715$, $x_{75} = 0.2$, $x_{78} = 0.1681$
12	0.03	0.0492	0.10	$x_1 = 0.03$, $x_{10} = 0.03$, $x_{14} = 0.0355$, $x_{21} = 0.03$, $x_{34} = 0.03$, $x_{40} = 0.0354$, $x_{47} = 0.03$, $x_{60} = 0.2$, $x_{61} = 0.1492$, $x_{69} = 0.03$, $x_{75} = 0.2$, $x_{78} = 0.2$
13	0.03	0.05	0.10	$x_1 = 0.03$, $x_{14} = 0.03$, $x_{16} = 0.0645$, $x_{22} = 0.03$, $x_{34} = 0.1829$, $x_{40} = 0.0560$, $x_{60} = 0.2$, $x_{61} = 0.03$, $x_{62} = 0.03$, $x_{69} = 0.0865$, $x_{72} = 0.03$, $x_{75} = 0.2$, $x_{82} = 0.03$
14	0.03	0.0494	0.10	$x_1 = 0.03$, $x_{14} = 0.03$, $x_{16} = 0.0748$, $x_{21} = 0.03$, $x_{34} = 0.2$, $x_{40} = 0.0336$, $x_{60} = 0.2$, $x_{61} = 0.03$, $x_{62} = 0.03$, $x_{69} = 0.0516$, $x_{72} = 0.03$, $x_{75} = 0.2$, $x_{78} = 0.03$, $x_{82} = 0.03$

Table 3 (*continue*)

	Objectives			Selected stocks
K	Return	Risk	Liquidity	
15	0.03	0.05	0.1012	$x_{10} = 0.03$, $x_{15} = 0.03$, $x_{32} = 0.0748$, $x_{34} = 0.03$, $x_{40} = 0.0306$, $x_{47} = 0.03$, $x_{60} = 0.0886$, $x_{61} = 0.03$, $x_{69} = 0.2$, $x_{72} = 0.03$, $x_{75} = 0.2$, $x_{77} = 0.03$, $x_{78} = 0.1808$, $x_{80} = 0.03$, $x_{85} = 0.03$

The first column in Table 3 shows the desired number of securities. Columns 2 to 4 contain the return, risk and liquidity of the optimal solutions while the last column shows the selected securities of the optimal portfolio. From this table, it can be seen that the return goal for all portfolios are completely satisfied which indicates that the target value set for this goal is realistic and achievable to the degree that there is no room for possible further improvement. The second goal of minimising the portfolio risk is fully achieved when the numbers of securities are 11, 13 and 15. However, the risk for portfolio with 10, 12 and 14 securities has been reduced by 0.0005, 0.0008 and 0.0006. Finally, if we vary the number of securities in a portfolio from 10 to 14, the liquidity goal is completely satisfied. For portfolio with 15 securities, the liquidity goal is shown to be about 0.0012, more than the target value of 0.10.

CONCLUSION

This paper is concerned with multi-objective portfolio optimisation problem in a fuzzy environment. Based on semi-absolute deviation as the risk measure, a goal programming model is proposed in which three objectives are optimised in a lexicographic order. The three objectives are: minimisation of portfolio risk, maximisation of portfolio return and maximisation of liquidity. Additionally, the model also considers three practical constraints which are cardinality, floor and ceiling constraints. A numerical example using real data from Bursa Malaysia is also presented to illustrate the modeling concept. The results indicate that the proposed model could generate satisfying portfolio selection strategies to investors.

Finally, future research may be conducted to investigate the impact of inclusion of other real-life constraints such as minimum transaction lot and sector diversification constraint in the model.

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