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General Formula for Calculating Accumulated Amount Based on Average Lowest Balance Concept

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ABSTRACT

One of the ways to calculate dividend for an investment is by using average lowest balance (ALB) concept. The existing calculation of dividend based on ALB concept can only be done yearly. This paper discusses on the development of a general formula to calculate the accumulated amount for any period of time, based on the ALB concept that considers different yearly dividend rates. The patterns for each variable and coefficient for the calculated yearly accumulated amount were analysed. The general forms of each variable and coefficient were then combined to form the general formula for calculating the accumulated amount. Validity of the general formula is confirmed by calculating the percentage errors and proven by using mathematical induction.

Keywords: Investment, average lowest balance, pattern analysing, general formula, accumulated amount

INTRODUCTION

In finance, the three most common formulas used to calculate dividend earned are simple interest, compound interest and annuity formulas. In annuity, equal amount of money is invested at equal interval of time over a certain period, where the rate of dividend is assumed to be the same throughout the period of investment. Annuity uses the concept of compound interest. In this paper, the assumption made is almost the same as in annuity, that is equal amount of money is invested at an equal interval of time. However, the yearly dividend rates are assumed to be different and the dividend calculated is based on the ALB concept.

In Malaysia, among the investment plans that use the ALB concept in calculating dividends are Tabung Haji (TH), Amanah

Saham Bumiputera (ASB) and Skim Simpanan Pendidikan Negara (SSPN-i) (Azlan, 2012; Zaaba, 2010). The current practice of finding the accumulated amount based on ALB concept is by adding up the lowest balance of each month in a year to obtain the average balance for the year. Then, the average balance will be used to find the dividend earned by using the simple interest formula. The accumulated amount obtained for the first year will be carried forward to the next year. The same calculation will be done for the second year, third year and so on. Thus, investment plans based on ALB cannot use annuity formula to find the accumulated amount. As of now, there is no general formula to calculate the accumulated amount over a period of time based on the ALB concept. This paper discusses the development of the general formula for calculating the accumulated amount based on ALB by observing the pattern of yearly dividend and accumulated amount earned.

Many researchers have developed mathematical models to solve problems in various fields such as in genetic study (Diehl & Görg, 2003), growing of cells (Finegood et al., 1995) and describing a real phenomenon (Nei & Li, 1979). While some other researchers from different fields such as music (Jones, 1987; Liu et al., 1999; Conklin, 2002; Meredith et al., 2002; Lu et al., 2004) and image processing (Jamil et al., 2004; Jamil & Bakar, 2006) analysed patterns to solve their problems. There are also some social studies done on investment of Tabung Haji (TH), Amanah Saham Bumiputera (ASB) and Skim Simpanan Pendidikan Negara (SSPN-i) (Zin, 1999; Ishak, 2011; Yusuf, 2011; Yahaya et al., 2009; Haron et al., 2013; Zainal et al., 2009; Hamzah et al., 2011; Musa et al., 2011). In this paper, a mathematical model is developed to solve a problem in the finance field. The method used is developing the model by analysing patterns.

In this study, the general formula is developed based on the existing calculations for calculating yearly accumulated amount. The calculation of dividend earned for the ith year, D_i is as follows:

$$D_i = P_i r_i t_i, \tag{1}$$

where P_i is the average lowest balance of each month in ith year, r_i is the dividend rate for the i_{th} year and t_i is the term (in year). The term P_i can be obtained by using (2):

$$P_i = \frac{\sum_{n=1}^{12} P_n}{12}, \tag{2}$$

where P_n is the monthly lowest balance in a year. After all the values of D_i have been calculated, the accumulated amount for mth year, S_m can then be obtained using (3), as follows.

$$S_m = \sum_{i=1}^{m} D_i + 12Bm, \tag{3}$$

where D_i is the dividend earned until the mth year, B is the monthly savings amount and m is the number of years.

The frequency of depositing money into the account, f, is assumed to be of monthly basis or equivalent to 12. Based on Azlan (2012), Zaaba (2010), and Diehl and Görg (2003), the equations for calculating yearly accumulated amount are as follows:

$$S_1 = B(12 + 6.5r_1) \tag{4}$$

$$S_2 = B(24 + 18.5r_2 + 6.5r_1(1 + r_2)) \tag{5}$$

$$S_3 = B(36 + 30.5r_3 + 18.5r_2(1 + r_3) + 6.5r_1(1 + r_2)(1 + r_3)) \tag{6}$$

$$S_4 = B(48 + 42.5r_4 + 30.5r_3(1 + r_4) + 18.5r_2(1 + r_3)(1 + r_4) + 6.5r_1(1 + r_2)(1 + r_3)(1 + r_4)) \tag{7}$$

Equations (4), (5), (6) and (7) are used as references in order to develop the general formula.

DEVELOPING THE GENERAL FORMULA

The General Form of the Variables

The development of the general formula begins with analysing the pattern of the variables. The variables for the second term until the last term of each equation (4), (5), (6) and (7) are put in a triangular coefficient, as shown in Figure 1.
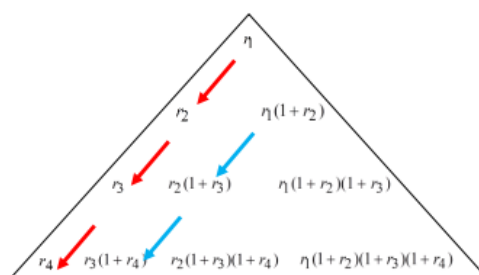

Figure 1. Triangular variable

The patterns of the variable for the second and the third terms are obvious, as shown by the red and blue arrows, respectively. Tables 1 and 2 show a summary of variables for the second and the third terms.

Table 1
A summary of the variables for the second term

Number of Year(s), m	S_m	Variable for Second Term
1	S_1	r_1
2	S_2	r_2
3	S_3	r_3
4	S_4	r_4

Table 2
Summary of variables for the third term

Number of Year(s), m	S_m	Variable for Second Term
2	S_2	$r_1(1+r_2)$
3	S_3	$r_2(1+r_3)$
4	S_4	$r_3(1+r_4)$

From Tables 1 and 2, the general form for the variable of the second term is r_m, where $m=1,2,3,...$ while the general form for the variable of the third term is $r_{m-1}(1+r_m)$ where $m=2,3,4,...$

Rearranging equation (7) is done to obtain the general formula for the variable of the fourth term and above.

$$S_4 = B(48 + 42.5r_4 + 30.5r_3(1+r_4) + 6.5r_1(1+r_2)(1+r_3)(1+r_4) + 18.5r_2(1+r_3)(1+r_4)) \quad (8)$$

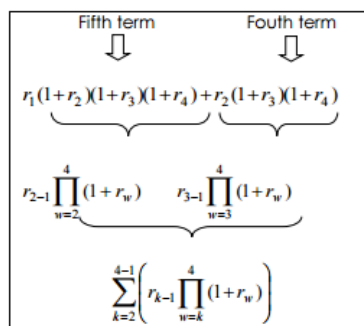Equation (8) can be visualised as in Figure 2.

Figure 2. A summation of fourth and fifth terms in S_4

Figure 2 explains the formation of general formula for variables of the fourth term and above. Even though the equation for calculating the accumulated amount has more than five terms, it was found that the pattern of summation was still the same as in Figure 2. Hence, the general formula for the variables of the fourth term and above is;

$$\sum_{k=2}^{m-1} \left(r_{k-1} \prod_{w=k}^{m} (1+r_w) \right) \quad \text{where} \quad m = 3,4,5,...$$

The General Formula for the Coefficients

The coefficients in equations (4), (5), (6) and (7) are arranged and can be visualised as follows:
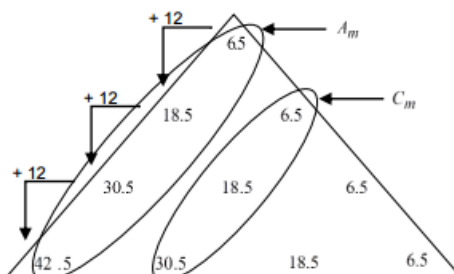

Figure 3. Triangular coefficient

In Figure 3, the value of the coefficient at the vertex is 6.5. This coefficient can be calculated based on the formula of average lowest balance for the first year, which is:

$$P_1 = \frac{\left(a^2 \sum_{y=1}^{f} y_n\right) - 12(a-1)}{12},$$

(9)

where P_i is the average lowest balance for the first year, f is the frequency of depositing money into the account and $a = \dfrac{12}{f}$. In the example, f is 12 and thus, $a=1$. Equation (9) is only suitable for f equals to 1, 2, 3, 4, 6 and 12 only. Next, the general formula for A_m is developed (refer to Table 3).

Table 3
Summary of variables for the third term

Number of Year(s), m	S_m	Coefficient for Second Term	General Form of Coefficient
1	S_1	6.5	$P_1+12(m-1)$
2	S_2	18.5	$P_1+12(m-1)$
3	S_3	30.5	$P_1+12(m-1)$
4	S_4	42.5	$P_1+12(m-1)$

It can be seen that the coefficient for the second term is the addition of P_1 and $12(m-1)$. This is shown in Figure 3, whereby the first coefficient of the following row is obtained by adding 12 to the coefficient of the previous row. Hence, the general formula for A_m is:

$$A_m = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(m-1),$$

(10)

As with A_m, the value of P_1 is still maintained, but it must be added with $12(m\text{-}2)$. Hence, the general formula for C_m is as follows:

$$C_m = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(m-2), \tag{11}$$

Previously, to form the general formula for the variables of the fourth term and above, the summation symbol was used. Since the coefficient of the fourth term and above must be added, the general form of these terms must be placed inside the summation, too. Hence, the general form of the fourth term and above must be written in terms of the variable k and not in terms of variable m. In the general formula of the variables for the fourth term and above, the index of summation started from $k=2$ until $k=m\text{-}1$. By relating the index k with the coefficient for the fourth and the fifth terms, the general form for the coefficients are summarised in Table 4.

Table 4
A summary of coefficient for fourth and the fifth terms

Sm	Value of k	Coefficient for Fourth Term	Coefficient for Fifth Term	General Form of Coefficient
S_3	6.5	-		$P_1+12(k\text{-}2)$
S_4			6.5	$P_1+12(k\text{-}2)$
	18.5			$P_1+12(k\text{-}2)$

Hence, from Table 4, the general form for the coefficients of the fourth terms and above is $P_1+12(k\text{-}2)$ and it is written as follows:

$$D_k = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(k-2), \tag{12}$$

Combining the General Formulas for Variables and Coefficients

The general formula for the first term, second term and the consecutive terms, as well as the coefficients for the terms can be combined to produce a general formula to calculate the accumulated amount. The general formula is as follows:

$$S_m = \begin{cases} B(12m + A_m r_m) \\ B(12m + A_m r_m + C_m r_{m-1}(1+r_m)) \\ B\left(\begin{array}{c} (12m + A_m r_m + C_m r_{m-1}(1+r_m)) \\ + \sum_{k=2}^{m-1}\left(D_k r_{k-1}\prod_{w=k}^{m}(1+r_w)\right) \end{array}\right) \end{cases} \tag{13}$$

Where,

$$A_m = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(m-1)$$

$$C_m = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(m-2)$$

$$D_k = \frac{\left(a^2 \sum_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(k-2)$$

Hence, the general formula for calculating the accumulated amount of money based on average monthly lowest balance that considers different yearly dividend rates has been obtained. Nonetheless, there are a few limitations to this formula. First, a person must practise discipline savings. Discipline saving refers to depositing the same amount of money at regular interval of time. In this study, a person may choose either to deposit money into the account every month, every two months, every three months, every four months, every six months or every twelve months. The frequency of depositing money into the account must be the same throughout the savings period amount. It is also assumed that no withdrawal is made during the savings period.

RESULTS AND DISCUSSION

The accuracy and reliability of the developed general formula is tested using percentage errors and mathematical induction.

Percentage Errors

The accumulated amount was calculated by using the general formulas that have been developed by using MAPLE. The results of calculations from MAPLE were then compared with the yearly calculations done using Microsoft Office Excel by finding the percentage error. The percentage errors were calculated for all the frequencies of depositing money into the account that was being considered in this study. Table 5 shows the percentage errors calculated in which the frequency is 12 (i.e., monthly deposits).

Table 5
The percentage error for monthly deposits

Year	Existing model (RM)	Developed model (RM)	Percentage error (%)
1	626.0000	626.0000	0
2	1306.8350	1306.8350	0
3	2029.2226	2029.2226	0
…			
7	5868.6116	5868.6117	1.70E-06
…			
15	17 526.4867	17 526.4867	0
16	19 420.7194	19 420.7195	5.15E-09
17	21 462.1570	21 462.1570	0
…			
23	37 695.8275	37 695.8275	0

From Table 5, most of the percentage errors are 0%. The largest error is 1.7×10^{-6}%, which is equivalent to 0.0000017% and it is considered to be a very small error that might be due to rounding off. Hence, the developed general formula can be considered as accurate and reliable.

Mathematical induction

According to Solow (2005), mathematical induction must be considered when a statement has the form "for every integer $m \geq 1$, something happens" The "something happens" is some statement $P(m)$, whereby it depends on the integer, m. Solow (2005) also mentioned that there are two words related to induction, which are, "integer" and also "≥ 1". Based on these statements, this study chose to prove the developed general formula by using the mathematical induction. Generally, there are two steps in induction, which are to verify that $P(1)$ is true and prove that $P(i+1)$ is true, assuming that $P(i)$ is true. Proving of the general formula (14) by using the mathematical induction is explained below:

For every integer $m \geq 3$,

$$S_m = B\left(12m + A_m r_m + C_m r_{m-1}(1+r_m) + \sum_{k=2}^{m-1}\left(D_k r_{k-1}\prod_{w=k}^{m}(1+r_w)\right)\right)$$

where

$$A_m = \frac{\left(a^2\sum_{y=1}^{f}y\right)-12(a-1)}{12}+12(m-1) = P_1 + 12(m-1)$$

$$C_m = \frac{\left(a^2 \sum\limits_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(m-2) = P_1 + 12(m-2)$$

$$D_k = \frac{\left(a^2 \sum\limits_{y=1}^{f} y\right) - 12(a-1)}{12} + 12(k-2) = P_1 + 12(k-2)$$

The analysis of the proof is discussed below. Let assume that the statement is written as follows:

$$P(m): \quad S_m = B\left(12m + A_m r_m + C_m r_{m-1}(1+r_m) + \sum_{k=2}^{m-1}(D_k r_{k-1} \prod_{w=k}^{m}(1+r_w))\right)$$

where

$$A_m = P_1 + 12(m-1)$$

$$C_m = P_1 + 12(m-2)$$

$$D_k = P_1 + 12(k-2)$$

The induction steps are as follows:

Step 1: Verify that $P(3)$ is true.

In this part, monthly deposits case was considered. Hence, $f=12$ and so $a=1$. All the values of m are replaced with 3. Therefore, by substituting these values into $P(m)$, $P(3)$ is obtained as follows:

$$P(3): S_3 = B(36 + 30.5r_3 + 18.5r_2(1+r_3) + 6.5r_1(1+r_2)(1+r_3))$$

Hence, the statement is clearly true for $m=3$.

Step 2: Assume $P(i)$ is true. By substituting m with i, $P(i)$ is as follows:

$$P(i): \quad S_i = B\left(12i + A_i r_i + C_i r_{i-1}(1+r_i) + \sum_{k=2}^{i-1}\left(D_k r_{k-1} \prod_{w=k}^{i}(1+r_w)\right)\right)$$

where

$$A_i = P_1 + 12(i-1)$$

$$C_i = P_1 + 12(i-2)$$

$$D_k = P_1 + 12(k-2)$$

Step 3: Show $P(i+1)$ is true. Then, $P(i+1)$ that needs to be obtained is as follows:

$$P(i+1): \quad S_{i+1} = B\left(12(i+1) + A_{i+1}r_{i+1} + C_{i+1}r_i(1+r_{i+1}) + \sum_{k=2}^{i}\left(D_k r_{k-1}\prod_{w=k}^{i+1}(1+r_w)\right)\right) \quad (15)$$

where

$$A_{i+1} = P_1 + 12i$$

$$C_{i+1} = P_1 + 12(i-1)$$

$$D_k = P_1 + 12(k-2)$$

In order to show that $P(i+1)$ is true, the left hand side (LHS) of (15) can be written as follows:

$$P(i+1): \quad S_{i+1} = S_i + [(S_i + BP_1)r_{i+1}] + 12B \quad (16)$$

The accumulated amount until the ith year is added with the dividend $i+1^{th}$ of the year and the total monthly savings for the $i+1^{th}$ year. The right hand side (RHS) of (16) is expanded and hence, $P(i+1)$ can be obtained as follows:

$$P(i+1): \quad S_{i+1} = S_i + [(S_i + BP_1)r_{i+1}] + 12B$$

$$= B\left[12i + A_i r_i + C_i r_{i-1}(1+r_i) + \sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w)\right)\right] + \left[B\left(\begin{array}{c}12i + A_i r_i + C_i r_{i-1}(1+r_i) + \\ \sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w))\right)\end{array}\right) + BP_1\right]r_{i+1} + 12B$$

$$= B\left[\begin{array}{c}12i + A_i r_i + C_i r_{i-1}(1+r_i) + \sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w)\right) + 12i r_{i+1} + A_i r_i r_{i+1} + C_i r_{i-1}r_{i+1}(1+r_i) \\ + (r_{i+1})\sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w)\right) + P_1 r_{i+1} + 12\end{array}\right]$$

$$= B\left[12(i+1) + (12i+P_1)r_{i+1} + A_i r_i(1+r_{i+1}) + C_i r_{i-1}(1+r_i)(1+r_{i+1}) + \left(\left(\sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w)\right)\right)(1+r_{i+1})\right.\right.$$

$$= \left[\begin{array}{c}12(i+1) + (12i + {}_1){}_{i+1} + ({}_1 + 12(i-1)){}_i(1+{}_{i+1}) + ({}_1 + 12(i-2)){}_{i-1}(1+{}_i)(1+{}_{i+1}) \\ + \left(\left(\sum_{k=2}^{i-1}\left({}_k {}_{k-1}\prod_{w=k}^{i}(1+{}_w)\right)\right)(1+{}_{i+1})\right)\end{array}\right]$$

$$= B\left[12(i+1) + A_{i+1}r_{i+1} + C_{i+1}r_i(1+r_{i+1}) + D_i r_{i-1}(1+r_i)(1+r_{i+1}) + \sum_{k=2}^{i-1}\left(D_k r_{k-1}\prod_{w=k}^{i}(1+r_w)(1+r_{i+1})\right)\right]$$

$$= B\left[12(i+1) + A_{i+1}r_{i+1} + C_{i+1}r_i(1+r_{i+1}) + \sum_{k=2}^{i}\left(D_k r_{k-1}\prod_{w=k}^{i+1}(1+r_w)\right)\right]$$

Therefore, equation (16) is equal to the right hand side of (15). Thus, the statement is true for $i+1$ and hence, it is true for all positive integers m. Therefore, the proof is complete.

CONCLUSION

The general formula for calculating the accumulated amount of money based on average monthly lowest balance at any period of time with different yearly dividend rates has been developed. Nonetheless, the general formula can only be used to calculate the accumulated amount of money if a person deposits the same amount of money into the account every month, every two months, every three months, every four months, every six months or every twelve months only.

The formula has been tested by using the MAPLE software. The accumulated amounts obtained from the calculations done using the MAPLE software were then compared with the accumulated amounts calculated using the existing formula. Comparisons were made by looking at the percentage error and the result shows that most of the percentage errors are 0%. Therefore, it can be concluded that the developed general formula gives accurate accumulated amount. Then, the general formula was proven by using mathematical induction. This formula can be used as an alternative general formula for calculating the accumulated amount as it is simple and easy to use. Finally, this is beneficial to the financial institutions that choose to adopt this concept of calculating dividends.

REFERENCES

Azlan, N. H. M. (2012). Calculations of dividend and accumulated amount for savings in Tabung Haji using triangular coefficient diagram. *Final Year Project B.Sc. (Hons) Mathematics*. UiTM Shah Alam, Malaysia.

Conklin, D. (2002). Representation and discovery of vertical patterns in music. In *Music and Artificial Intelligence* (pp. 32-42). Springer Berlin Heidelberg.

Diehl, S., & Görg, C. (2003). Experiencing natural phenomena with virtual, constructed and mathematical models. *Simulation Conference, Proceedings of the 2003 Winter* (pp. 778-781). IEEE.

Finegood, D. T, Scaglia, L., & Bonner-Weir, S. (1995). Dynamics of β-cell mass in the growing rat pancreas: Estimation with a simple mathematical model. *Diabetes, 44*(3), 249-256.

Hamzah, H., Musa, R., Muda, M., & Mohamed, R. N. (2011). Analysing the predicting factors influencing parents to save in National Education Savings Scheme [SSPN]. *Humanities, Science and Engineering (CHUSER), IEEE Colloquium* (pp. 951-956). IEEE.

Haron, S. A, Sharpe, D. L., Abdel-Ghany, M., & Masud, J. (2013). Moving up the savings hierarchy: Examining savings motives of older Malay Muslim. *Journal of Family and Economic Issues, 34*(3), 314-328.

Ishak, M. S. B. H. (2011). Tabung Haji as an Islamic Financial Institution for sustainable economic development. In *2nd International Conference on Humanities, Historical and Social Sciences IPEDR*. IACSIT Press, Singapore.

Jamil, N., & Bakar, Z. A. (2006). Shape-based image retrieval of songket motifs. In *19th Annual Conference of the NACCQ* (pp. 213-219). Wellington, New Zealand.

Jamil, N., Bakar, Z. A., & Sembok, T. M. T. (2004). A comparison of noise removal techniques in songket motif images. *Computer Graphics, Imaging and Visualisation, Proceedings, International Conference* (pp. 139-143).IEEE.

Jones, M. R. (1987). Dynamic pattern structure in music: Recent theory and research, *Perception and Psychophysics, 41*(6), 621-634.

Liu, C. C., Hsu, J. L., & Chen, A. L. (1999). Efficient theme and non-trivial repeating pattern discovering music in databases. *Data Engineering, Proceedings, 15th International Conference* (pp. 14-21). IEEE.

Lu, L., Wang, M., & Zhang, H. J. (2004). Repeating pattern discovery and structure analysis from acoustic music data. In *Proceedings of the 6thACM SIGMM International Workshop on Multimedia Information Retrieval* (pp. 275-282). ACM.

Meredith, D., Lemström, K., & Wiggins, G. A. (2002). Algorithms for discovering repeated patterns in multidimensional representations of polyphonic music. *Journal of New Music Research, 31*(4), 321-345.

Musa, R., Lennora, P., & Hamzah, H. (2011). *Precautionary saving intention in SSPN by Malaysian parents: Theory of planned behaviour approach*. Research Management Institute of UiTM Shah Alam.

Nei, M., & Li, W. H. (1979). Mathematical model for studying genetic variation in terms of restriction endonucleases. *Proceedings of the National Academy of Sciences, 76*(10), 5269-5273.

Solow, D. (2005). *How to read and do proofs* (4th Ed.). Cleveland, OH: John Wiley & Sons Inc.

Yahaya, A., Yahaya, N., Ramli, J., Boon, Y., & Abd Ghaffar, M. (2009). Amanah Saham National Berhad's: Promotional strategies and its relationship with customer motivation. *Journal of Social Sciences, 5*(4), 283-290.

Yusuf, M. B. (2011). Effects of spiritual capital on Muslim economy: The case of Malaysia. *Research on Humanities and Social Sciences, 1*(2), 23-40.

Zaaba, F. S. (2010). The mathematical model for education saving plan in National Education Savings Scheme [SSPN]. *Final Year Project B.Sc. (Hons) Mathematics*. UiTM Shah Alam, Malaysia.

Zainal, N. R., Kamaruddin, R., & Nathan, S. B. S. (2009). Socio-Economic Status and Parental Savings for Higher Education among Malaysian Bumiputera Families. *International Journal of conomics and Finance, 1*(2), 170-173.

Zin, R. H. M. (1999). Malaysian reverse investments: Trends and strategies. *Asia Pacific Journal of Management, 16*(3), 469-496.